# Automatic Compact Modelling for MEMS: Applications, Methods and Tools

## Lecture 1: Introduction to Dynamic Systems and Model Reduction

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**http://www.imtek.uni-freiburg.de/simulation/mor4ansys/**

IMTEK

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# Outline

- Introduction
- Compact Modeling vs. Model Reduction
- Dynamic System
- Model Reduction Overview

Forging a smaller System



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# Prof Korvink's Simulation Lab

- **MEMS/MST**
- **Knowledge of engineering needs.**
- **Following modern numerical methods.**
- **Better tools for engineers:**
  - ✓Design,
  - ✓System-level simulation.

- **Chair of Simulation**
- **http://www.imtek.de/simulation/**



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# MOR Research at IMTEK

- **2001:**
  - ✓Micropyros Project.
- **2002:**
  - ✓Review on model reduction options.
  - ✓Prototyping in Mathematica.
- **2003:**
  - ✓mor4ansys for a thermal problem.
- **2004:**
  - ✓mor4ansys for structural mechanics.

- **MOR for ANSYS website:**
  - ✓9 journal papers;
  - ✓1 book chapter;
  - ✓1 PhD thesis;
  - ✓33 papers in conference proceedings.
- **Collaboration:**
  - ✓Freescale, Germany;
  - ✓IMEGO, Sweden;
  - ✓Sensirion, Switzerland;
  - ✓Phillips, Netherlands.

# Course Goals

- **To understand the technology:**
  - ✓Hierarchy of model reduction method.
  - ✓Linear model reduction.
  - ✓Parameter-preserving model reduction.
  - ✓Nonlinear model reduction.
- **To distinguish between different levels:**
  - ✓It can be already used in routine work.
  - ✓Research is still required.
  - ✓At the frontiers of science.
- **To learn what is possible:**
  - ✓Model reduction is not ubiquitous.
  - ✓Yet, there are many scenarios where it is working right now.
- **Software:**
  - ✓Main stress on practical things.

# Course Plan

- **8:30 - 9:20:**
  - ✓Introduction to Dynamic Systems and Model Reduction.
- **5 min break.**
- **9:25 - 10:20:**
  - ✓Implicit Moment Matching via Arnoldi Process: Theory.
- **20 min break**
- **10:40 - 11:30:**
  - ✓Implicit Moment Matching via Arnoldi Process: Practice.
- **5 min break**
- **11:35 - 12:30**
  - ✓Advanced topics.

# Software

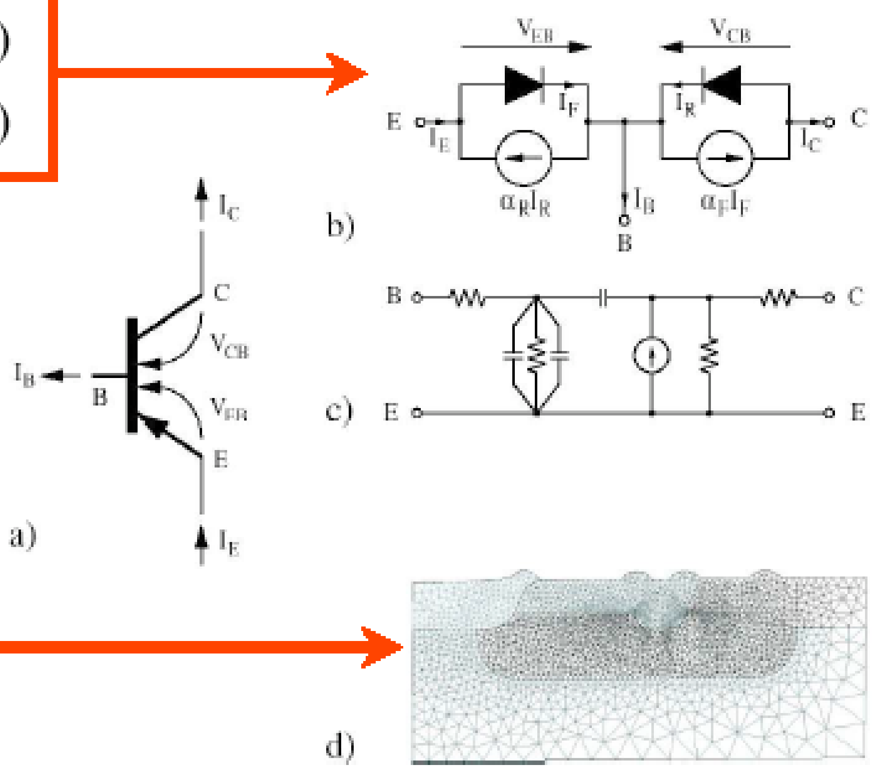
- **SLICOT (http://www.slicot.de/):**
  - ✓Model reduction methods from control theory.
  - ✓Free for research.
  - ✓MATLAB has licensed SLICOT.
- **MOR for ANSYS (former mor4ansys):**
  - ✓Implicit moment matching via the Arnoldi process.
  - ✓GNU Public License.
- **Mathematica functions:**
  - ✓Post4MOR to work with a reduced model.
  - ✓Mathlink interface to SLICOT.
  - ✓Mathlink interface to DOT optimizer.
  - ✓GNU Public License.

# Transistor Compact Model

$$I_E = I_{F0}(e^{qV_{EB}/kT} - 1) - \alpha_R I_{R0}(e^{qV_{CB}/kT} - 1)$$
$$I_E = \alpha_F I_{F0}(e^{qV_{EB}/kT} - 1) - I_{R0}(e^{qV_{CB}/kT} - 1)$$

Too much reliance on intuition

$$-\varepsilon\nabla^2\Psi = q(p - n + N_0)$$
$$\frac{\partial n}{\partial t} = \nabla \bullet (-\mu_n n\nabla\Psi + D_n\nabla n) - R_n$$
$$\frac{\partial p}{\partial t} = \nabla \bullet (\ \mu_p p\nabla\Psi + D_p\nabla p) - R_p$$



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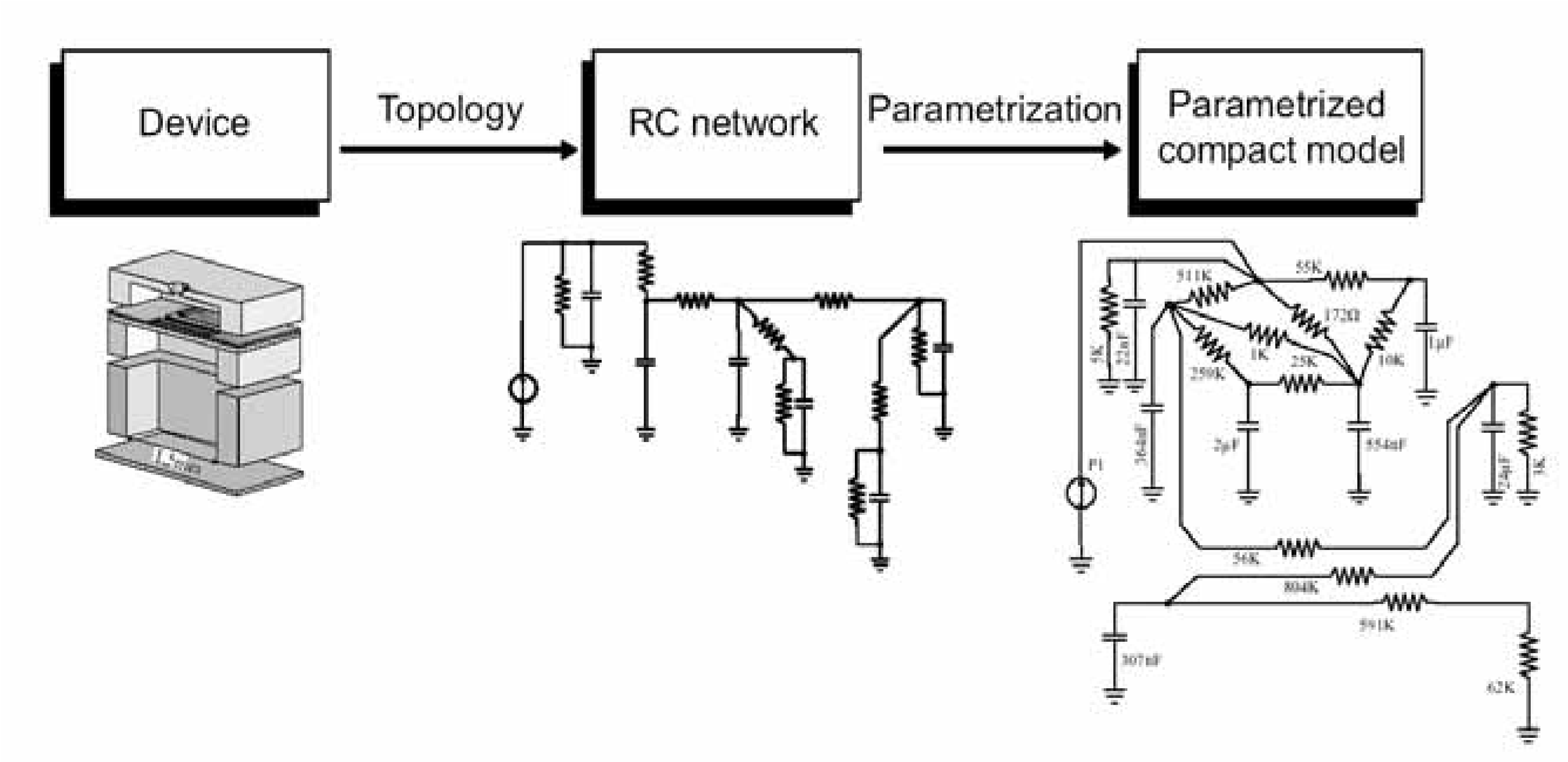
# Compact Thermal Models



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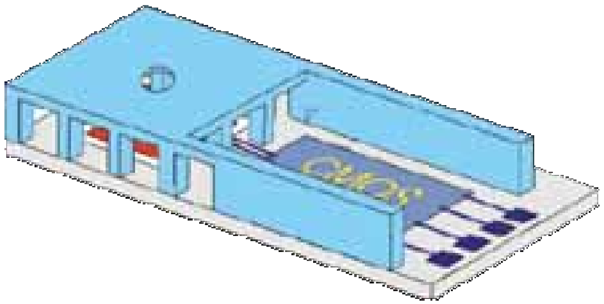
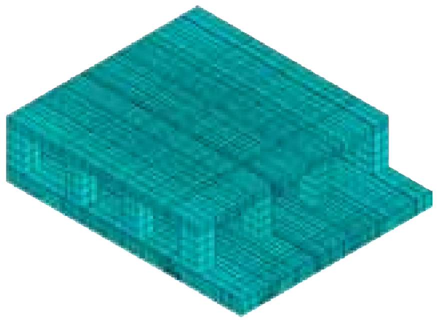
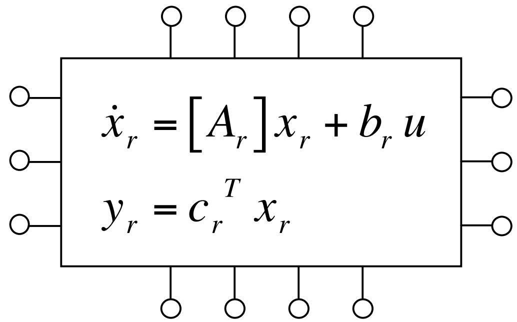
# Model Order Reduction

Physics & Geometry → **FEM** → System of $n$ ODEs → **MOR** → Reduced System of $r << n$ ODEs







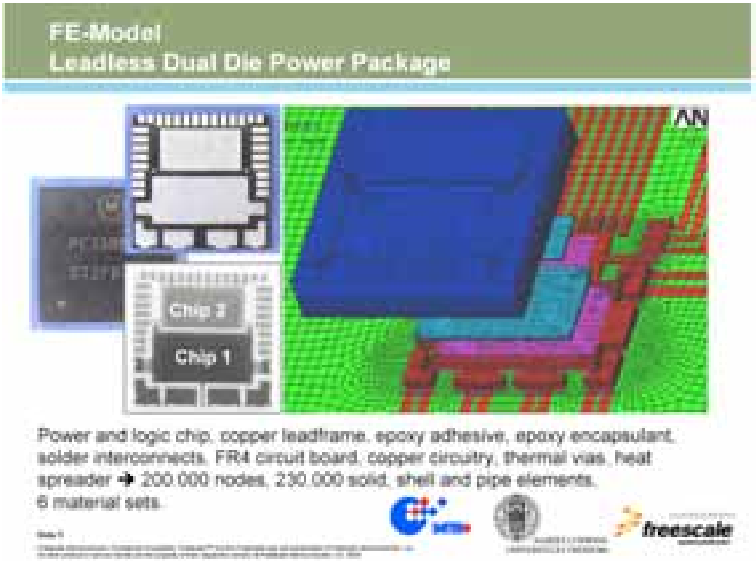

Order reduction is an efficient means to enable a system-level simulation

# Compact Modeling vs. Model Reduction

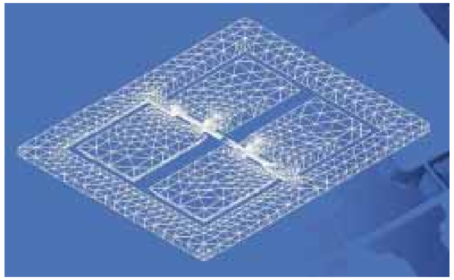
| *Method Properties* | *Compact Modeling* | *Model Reduction* |
|---|---|---|
| Reduced model: | Topology obtained by intuition | Formally obtained |
| Simulation of the original model: | Necessary | Not necessary |
| Experimental results: | Can be used | Cannot be used |
| Parameter extraction: | Necessary | Not necessary |
| System matrices: | Not used | Used |

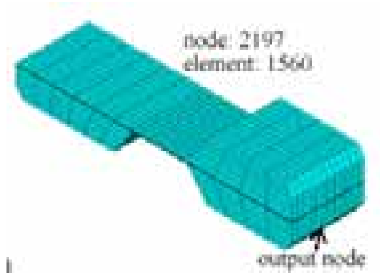
# Gallery of Examples I

- **Electro-Thermal**
  - ✓Constant material properties.
  - ✓Resistivity can depend on temperature.
  - ✓Preserving film coefficients in the symbolic form.
  - ✓Nonlinear film coefficients.

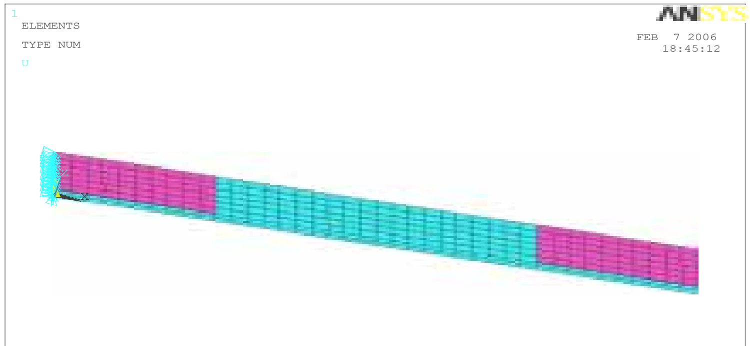



- **Structural Mechanics**
  - ✓Small deformations.



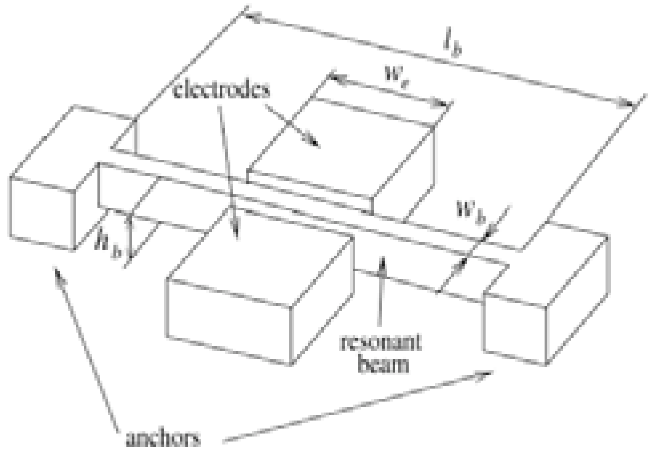


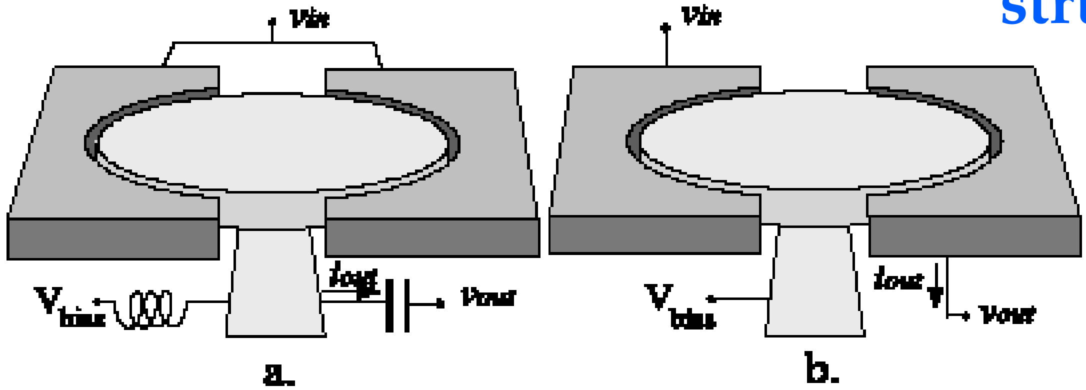

- **Piezoelectric actuators for control:**



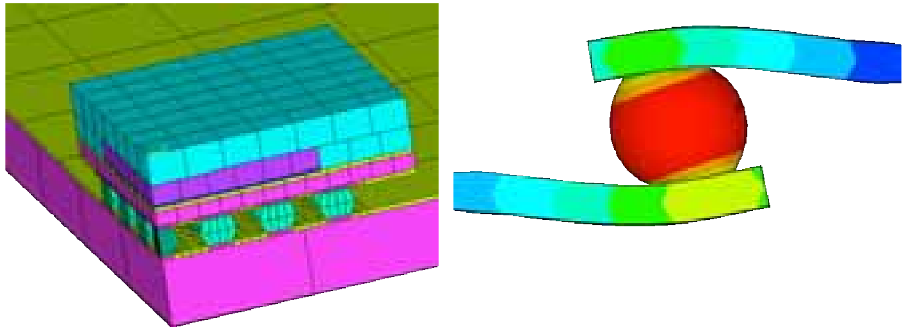
# Gallery of Examples II

- Pre-stressed small-signal analysis for RF-MEMS

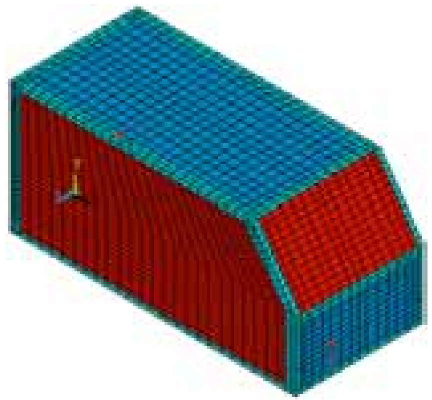



- Thermomechanical



- Acoustics including fluid-structure interactions





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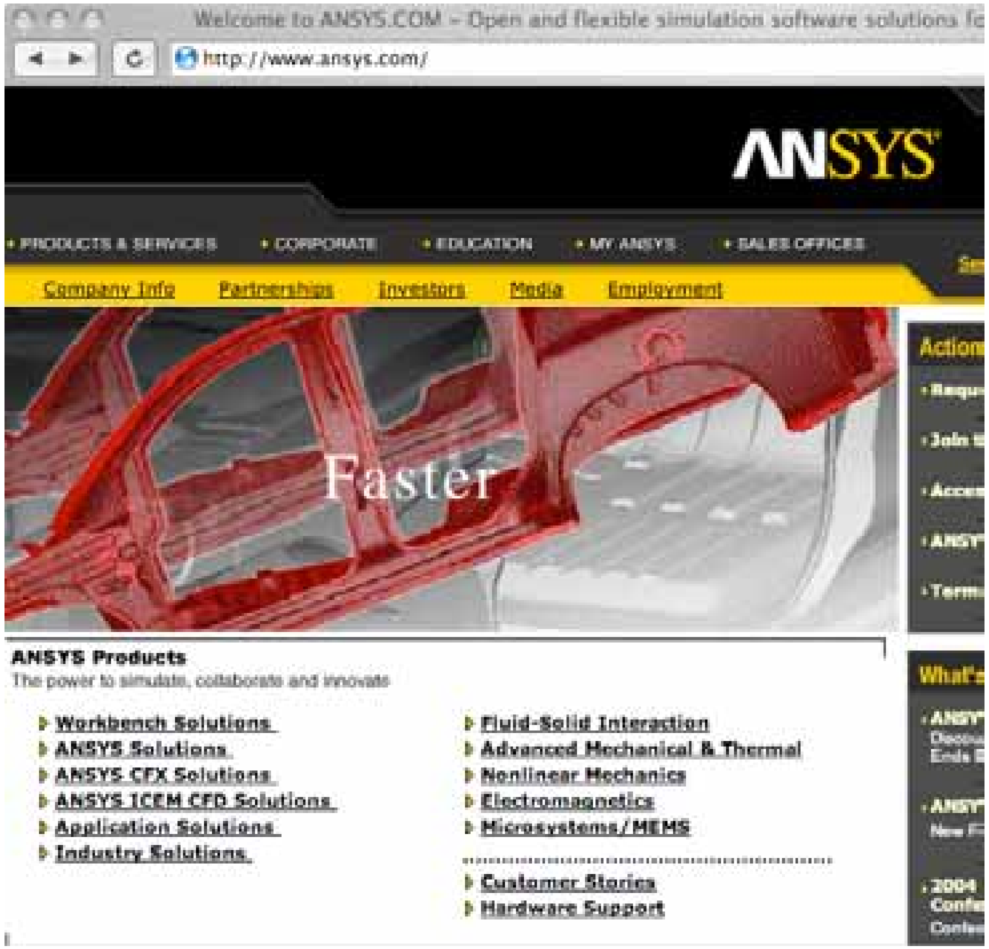
# Finite Element Method

- Commercial FEM software available.
- Multiphysics:
  - ✓Structural mechanics,
  - ✓Thermal,
  - ✓Fluidic,
- Transient and harmonic simulation takes too much time.
- Incompatible with system level simulation.



# From PDE to ODEs

- **Partial differential equation:**

$$\nabla(\kappa \nabla T) + Q - \rho C_p \frac{\partial T}{\partial t} = 0 \qquad \frac{\partial(\rho u S)}{\partial t} + \frac{\partial[(\rho u^2 + p)S]}{\partial x} = p\frac{dS}{dx}$$

- **Discretization methods:**
  - ✓Finite element method,
  - ✓Finite difference method,
  - ✓Finite volume method,
  - ✓Boundary element method.
- **Ordinary differential equations:**

$$E\frac{dx}{dt} + Kx = f \qquad M\frac{d^2x}{dt^2} + E\frac{dx}{dt} + Kx = f$$

# Dynamic System Notation

- **System matrices are constant:**
  - ✓linear ODEs.
- **Otherwise:**
  - ✓nonlinear ODEs.
- **First Order Explicit Form:**
  - ✓State-space model

$$\dot{x} = Ax + f$$

- **First Order Implicit Form:**
  - ✓control theory

$$E\dot{x} = Ax + f$$

  - ✓finite elements

$$E\dot{x} + Kx = f \qquad K = -A$$

- **Differential Algebraic Equations (DAE):**

$$\det(E) = 0$$

- **Second Order System:**

$$M\ddot{x} + E\dot{x} + Kx = f$$

- **Damping (heat conductivity) matrix is E (not C)**

# Transformation to State-Space Form

- **Implicit System:**

$$\dot{x} = E^{-1}Ax + E^{-1}f \qquad \dot{x} = -E^{-1}Kx + E^{-1}f$$

- **Second Order System:**

$$\begin{pmatrix} M & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} \dot{v} \\ \dot{x} \end{pmatrix} + \begin{pmatrix} E & K \\ -I & 0 \end{pmatrix}\begin{pmatrix} v \\ x \end{pmatrix} = \begin{pmatrix} f \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \dot{v} \\ \dot{x} \end{pmatrix} = \begin{pmatrix} -M^{-1}E & -M^{-1}K \\ I & 0 \end{pmatrix}\begin{pmatrix} v \\ x \end{pmatrix} + \begin{pmatrix} M^{-1}f \\ 0 \end{pmatrix}$$

- **DAEs can be transformed to ODEs by eliminating constraints for first derivatives.**

# Inputs and Outputs

- Split load vector to a constant part and input functions.
- Single input:
  - ✓vector by scalar input function

$$f(t) = bu(t)$$

- Multiple inputs:
  - ✓matrix by a vector of scalar input functions

$$f(t) = Bu(t) = \sum_i b_i u_i(t)$$

- We are interested in just some linear combination of the state vector:

$$y = Cx$$

- Single Input - Single Output (SISO).
- Multiple Inputs - Multiple Outputs (MIMO).
- Single Input - Complete Output (SICO).



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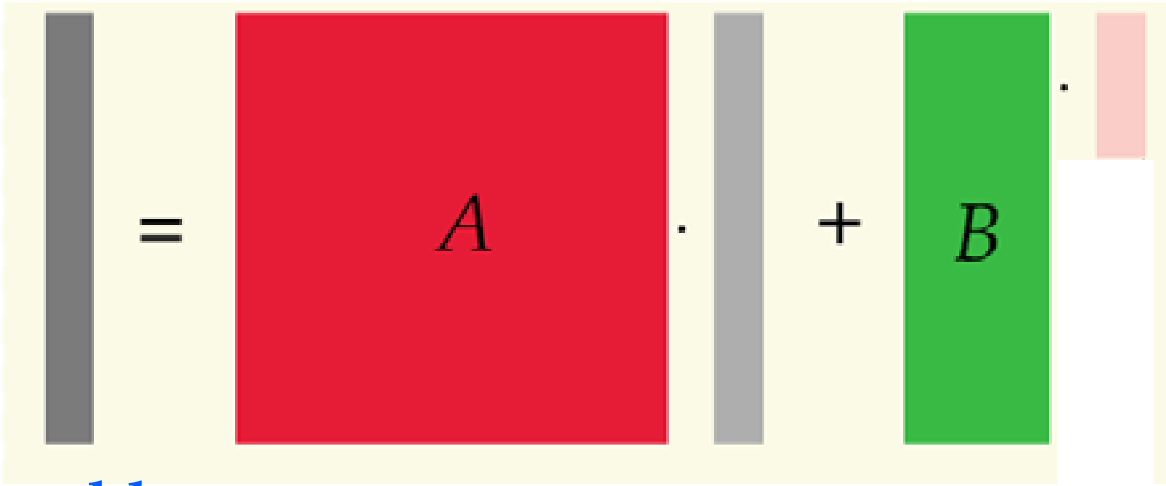
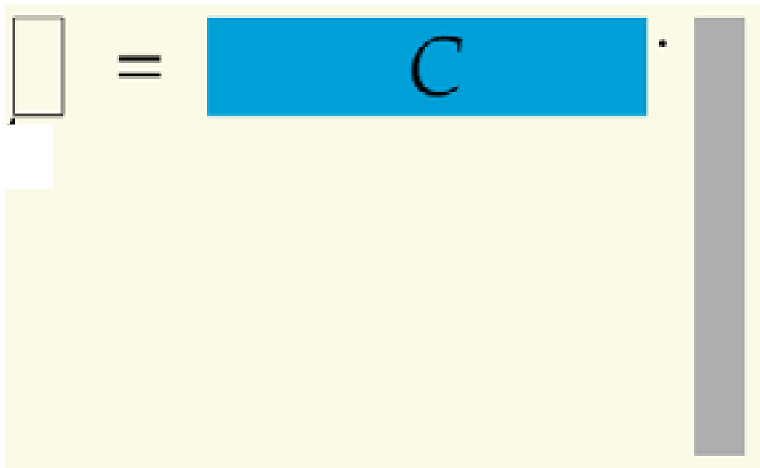
# Pictorial Form

$$\dot{x} = Ax + Bu$$
$$y = Cx$$




- **Number of inputs should be limited.**
- **Input functions do not take part in model reduction.**
- **Complete output is possible.**



# Static (stationary) Problem

- **The simplest problem:**

$$Kx = f$$

- **Basic computational block.**
- **Examples from ANSYS:**
  - ✓time in seconds,
  - ✓Sun Ultra 450 MHz 4 Gb.
- **Model reduction does not make sense.**

| Dimension | Time is ANSYS 8.1 |
|---:|---:|
| 4 267 | 0.63 |
| 11 445 | 2.2 |
| 20 360 | 15 |
| 79 171 | 230 |
| 152 943 | 95 |
| 180 597 | 150 |
| 375 801 | 490 |

# Transient Simulation

$$E\dot{x} + Kx = f$$

$$x(0) = x_0$$

$$x_{n+1} = x_n + \Delta t \cdot \dot{x}_{n+1}$$

$$(E/\Delta t + K)x_{n+1} = f + Ex_n/\Delta t$$

- **Initial value problem.**
- **Numerical integration (for example, backward Euler).**
- **Computational time is roughly static analysis time by a number of timesteps.**
- **Time can be reduced provided the timestep is fixed.**

# Time and Laplace Domains

- **Time domain:**
  - ✓zero initial conditions.

$$E\dot{x}(t) + Kx(t) = Bu(t)$$
$$y(t) = Cx(t)$$
$$x(0) = 0$$

- **Laplace transform:**
  - ✓s is a complex variable.

$$L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$$

- **Laplace domain):**

$$EsX(s) + KX(s) = BU(s)$$
$$Y(s) = CX(s)$$

- **Transfer function:**
  - ✓Low-dimensional matrix;
  - ✓Yet, the complexity to compute is high;
  - ✓Inverse of high dimensional matrix.

$$H(s) = \frac{Y(s)}{U(s)} = C(sE + K)^{-1}B$$

- **Bode plot:** $\omega = 2\pi f$

$$H(i\omega) = C(i\omega E + K)^{-1}B$$

# Harmonic Response Analysis

- **Assume harmonic input function:**

$$u(t) = u_o e^{i\omega t} = u_o\{\cos(\omega t) + i\sin(\omega t)\}$$

- **Assume harmonic response but with different phase angle:**

$$x(t) = x_o e^{i\varphi} e^{i\omega t} = x_o\{\cos(\varphi) + i\sin(\varphi)\}e^{i\omega t} =$$
$$= (x_{o,re} + ix_{o,im})e^{i\omega t}$$

- **Substitute into a dynamic system:**

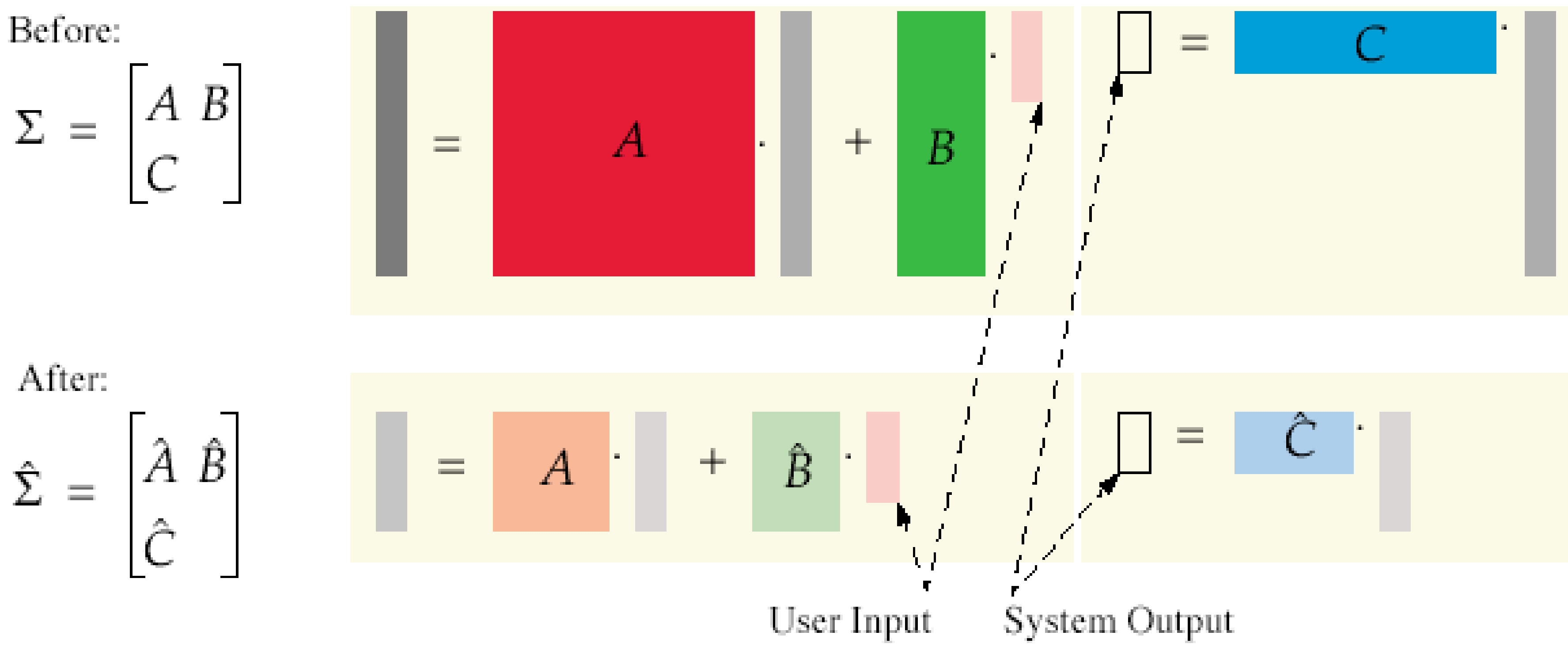
$$(i\omega E + K)(x_{o,re} + ix_{o,im})e^{i\omega t} = Bu_o e^{i\omega t}$$

- **Harmonic simulation:**
  - ✓Computation time is roughly static analysis time by a number of frequencies.

$$(i\omega E + K)x(\omega) = Bu_o$$
$$y(\omega) = Cx(\omega)$$

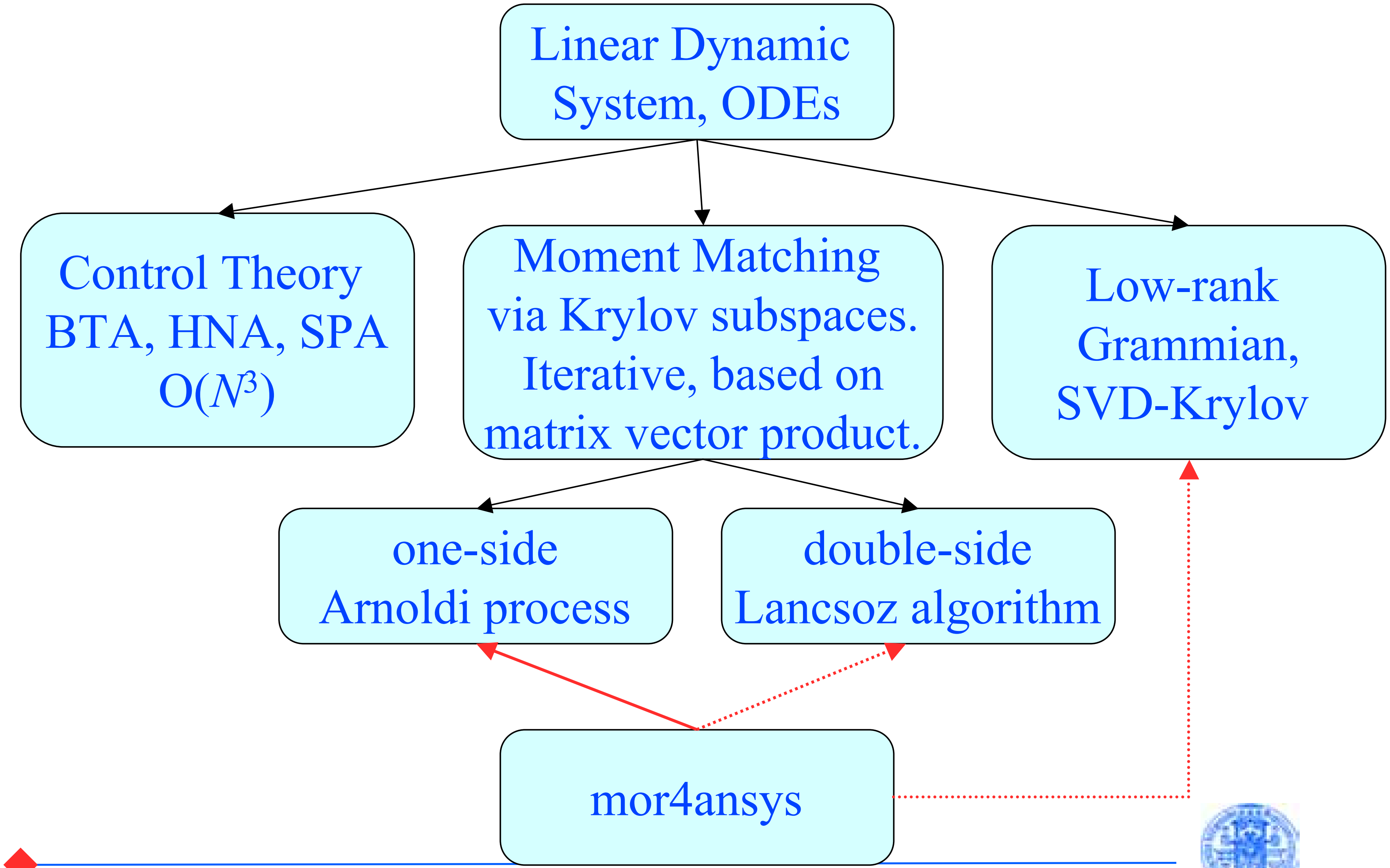
$$y(\omega) = C(i\omega E + K)^{-1} Bu_0$$

# Model Reduction




- **Approximation of Large-Scale Dynamic Systems**
  - ✓Prof. Antoulas (http://www-ece.rice.edu/~aca/)

# Linear Model Reduction Methods



# Linear Model Reduction Methods

| | ☺ Advantages | ☹ Disadvantages |
|---|---|---|
| Control theory methods | **global error estimate, fully automatic** | **computational effort $O(n^3)$** |
| Padé approximants | **low computational effort** | **no global error estimate, manual selection of $r$** |
| SVD-Krylov | **global error estimate, computational effort < $O(n^2)$** | **currently under development** |
| Guyan-based methods | **preserve the physical nodes** | **unnecessarily large reduced order models** |

# Summary

- Introduction
- Compact Modeling vs. Model Reduction
- Dynamic System
- Model Reduction Overview



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