Automatic Compact Modelling for MEMS: Applications, Methods and Tools

Lecture 3:

Implicit Moment Matching via Arnoldi Process: Practice

Evgenii B. Rudnyi, Jan G. Korvink

http://www.imtek.uni-freiburg.de/simulation/mor4ansys/

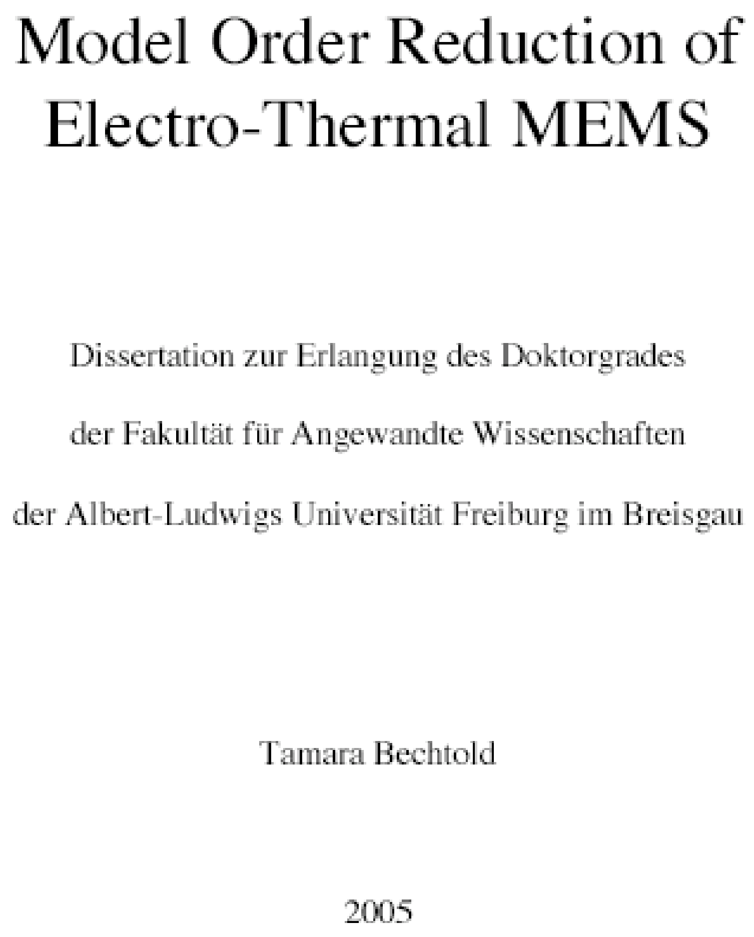
IMTEK

ALBERT-LUDWIGS-
UNIVERSITÄT FREIBURG

Outline

- **Electro-thermal MEMS (Tamara Bechtold)**
- **Structural mechanics**
  - ✓Impeller model (Prof Takano, Ritsumeikan)
  - ✓Geometry optimization of an accelerometer (Prof Han, ANU)
- **Piezoelectric actuators for control (Soong-Oh Han, Darmstadt)**
- **Pre-stressed small-signal analysis for RF-MEMS (Laura Del Tin)**
- **Thermomechanical models (Elena Zukowski)**
- **Acoustics including fluid-structure interactions (Srinivas Puri, Oxford)**

Tamara Bechtold



Model Order Reduction of
Electro-Thermal MEMS

Dissertation zur Erlangung des Doktorgrades
der Fakultät für Angewandte Wissenschaften
der Albert-Ludwigs Universität Freiburg im Breisgau

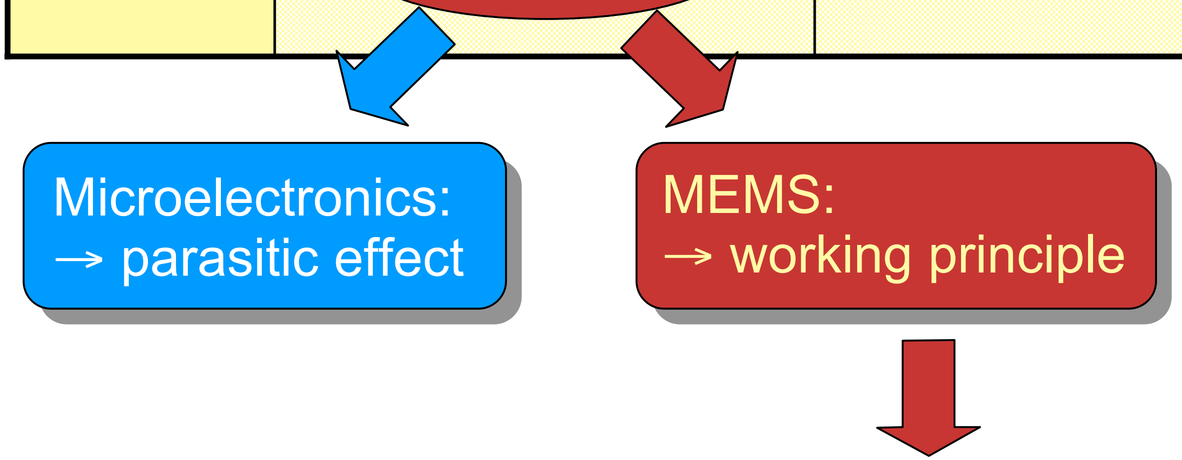
Tamara Bechtold

2005

- Thesis available at MOR for ANSYS site.
- Next slides are from her defense (slightly different notation).
- Tamara.Bechtold@philips.com

MEMS and Electro-Thermal Simulation

	Thermal	Electrical
Thermal	**Heat conduction**	**Thermoresistance**
Electrical	Joule heating	**Electrical conduction**



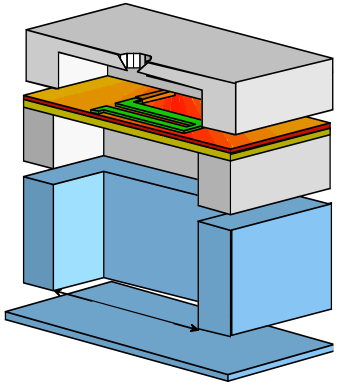
Microelectronics:
→ parasitic effect

MEMS:
→ working principle

- **microhotplate-based devices**
- electro-thermal actuators
- microfluidic devices

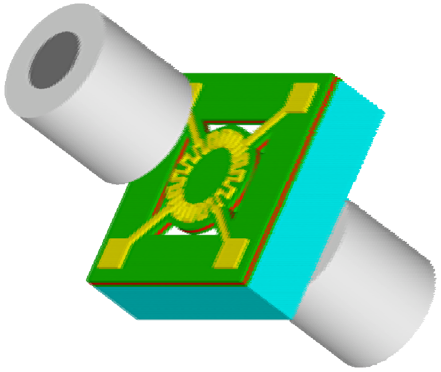
Electro-Thermal MEMS Case Studies

Microthruster

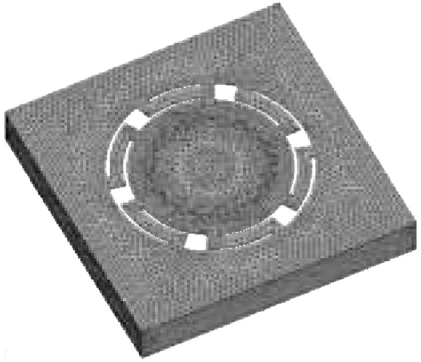


(Illustration courtesy of
C. Rossi, LAAS-CNRS)

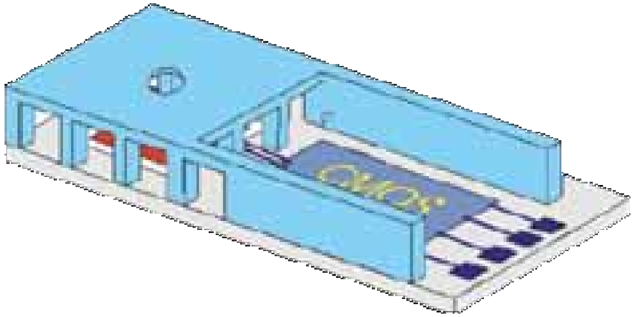
Tunable optical filter



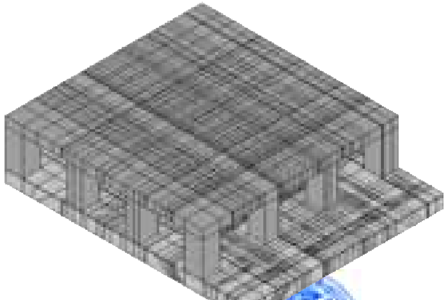
(Illustration courtesy of
D. Hohlfeld, IMTEK)



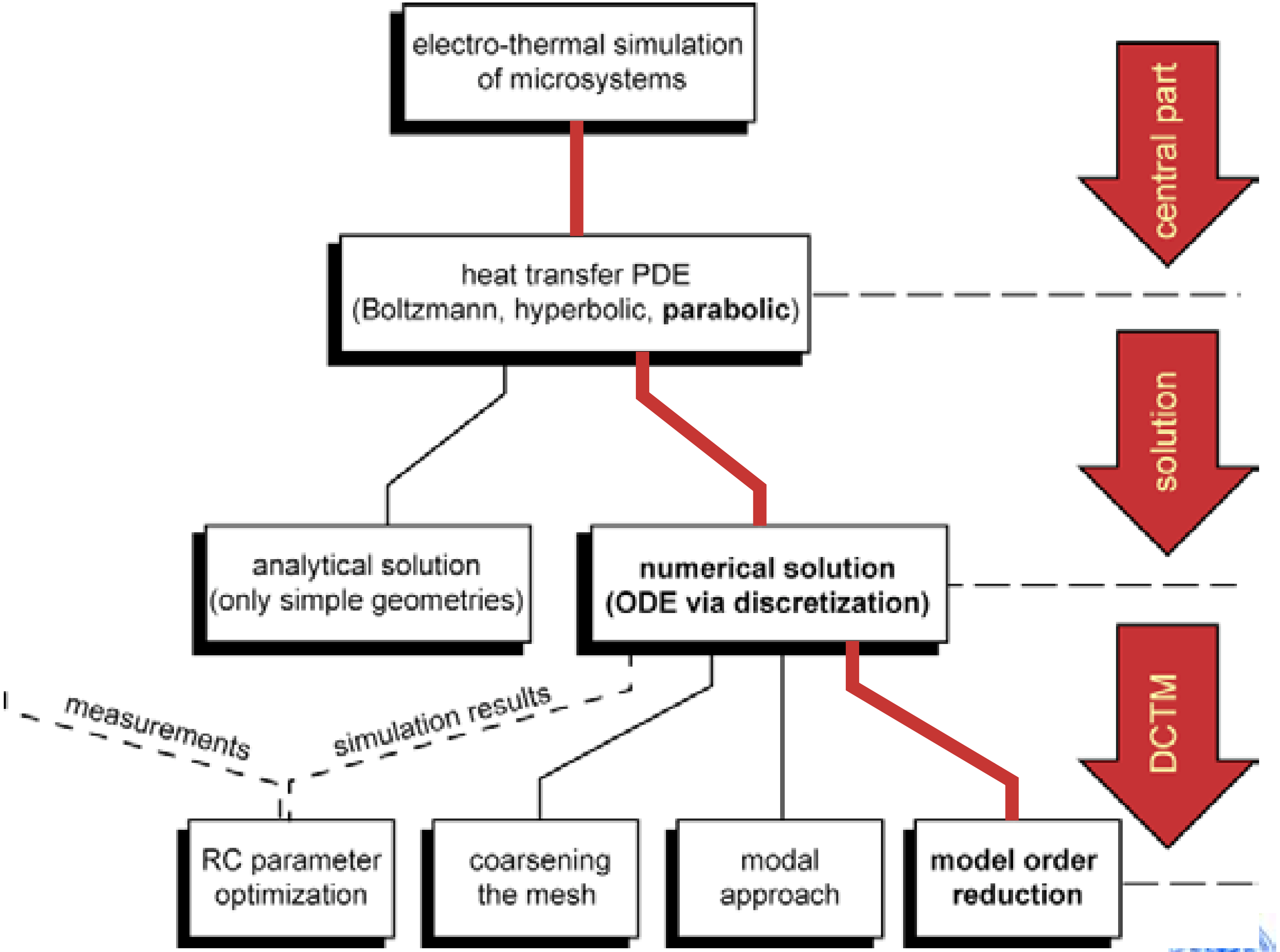
Gas sensor



(Illustration courtesy of
J. Wöllenstein, FhG IPM)



Research Path



electro-thermal simulation of microsystems

heat transfer PDE (Boltzmann, hyperbolic, **parabolic**)

analytical solution (only simple geometries)

numerical solution (ODE via discretization)

RC parameter optimization

coarsening the mesh

modal approach

model order reduction

Heat Transfer Equation

PDE:

$$\nabla(\kappa\,\nabla T) + Q - \rho\,C_p\,\frac{\partial T}{\partial t} = 0$$

spatial discretization

ODE:

$$C\dot{T} + KT = \sum f_{sources} + \sum f_{BC}$$

thermal BC:

$$\forall r \in \Omega:$$

$$t = 0, \quad T(r,t) = T_0(r)$$

$$\forall r \in \partial\Omega:$$

$$T(r) = T_{prescribed}(t)$$

$$q_\perp(r) = q_{prescribed}(t)$$

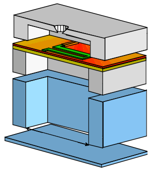
$$q_\perp(r) = h \cdot (T - T_{amb})$$

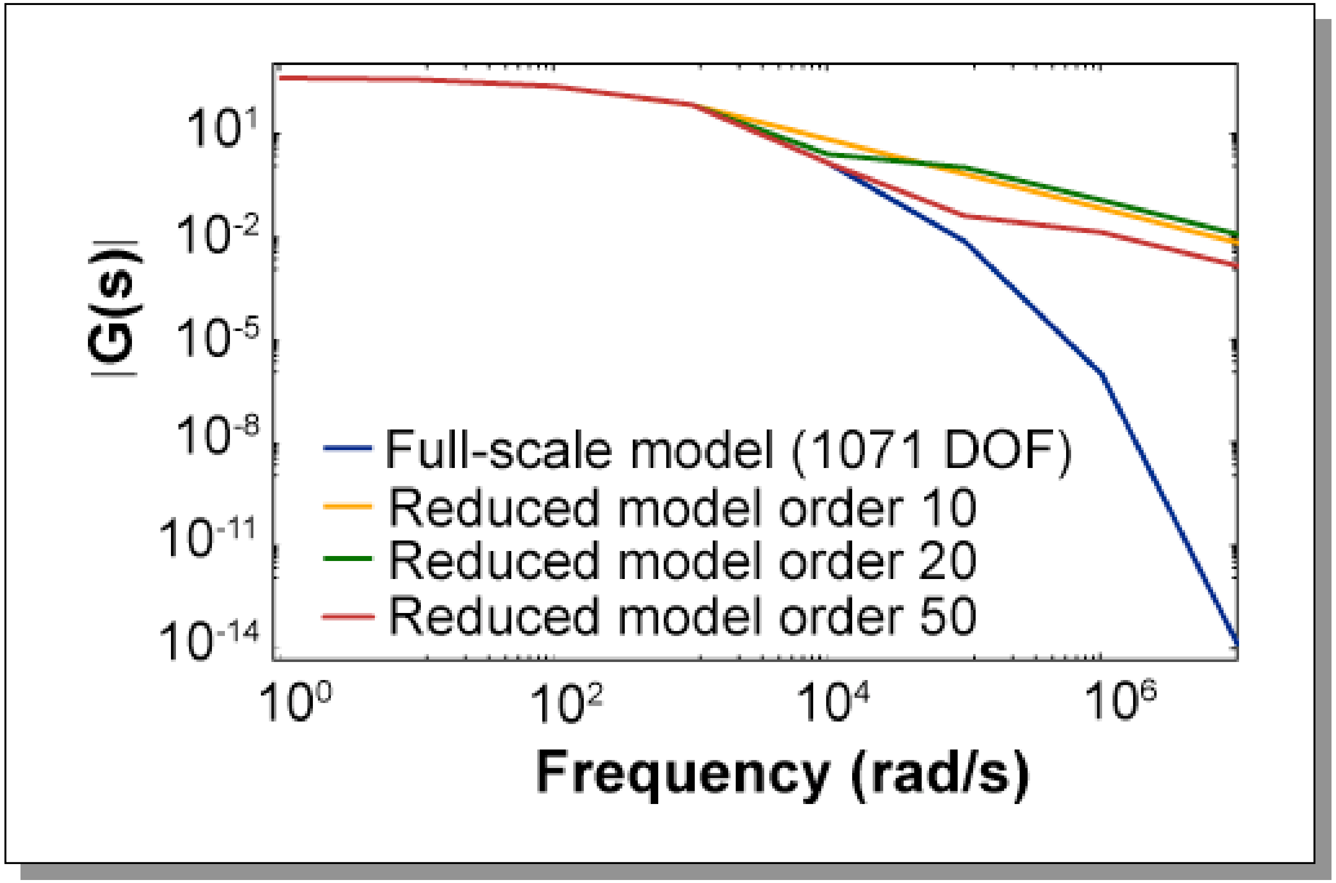
$$q_{emitted}(r) = \varepsilon\sigma A(T_{surf}^4 - T_{amb}^4)$$

suitable dimensions for accurate MEMS models ≥ 100,000 nodes

need for dynamic compact thermal modeling (DCTM)

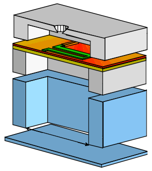
Frequency Response

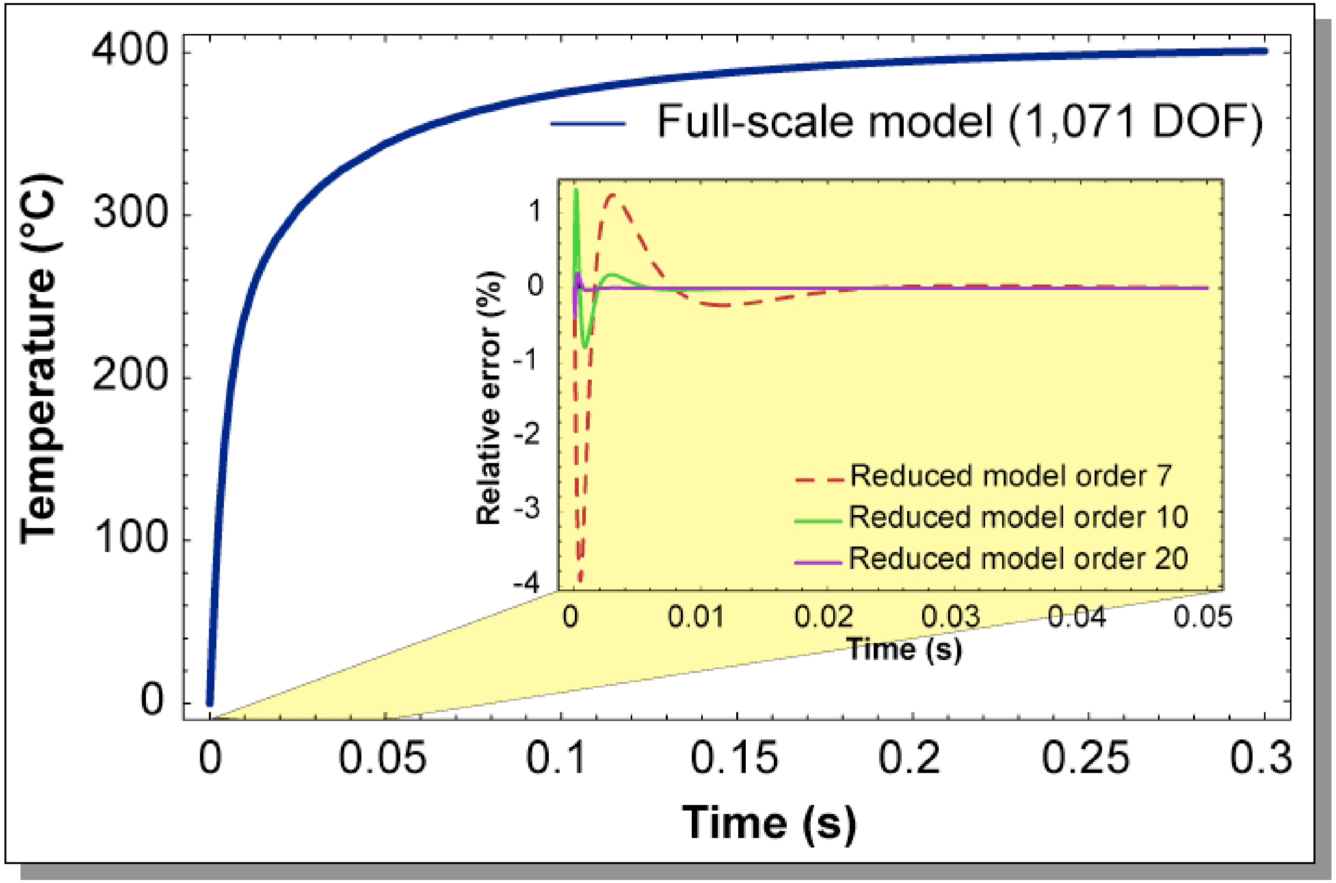




Good match in the frequency domain around expansion point: $s_0=0$

Step Response





Step response error almost vanishes within the initial 0.05s

Approximating Complete Output

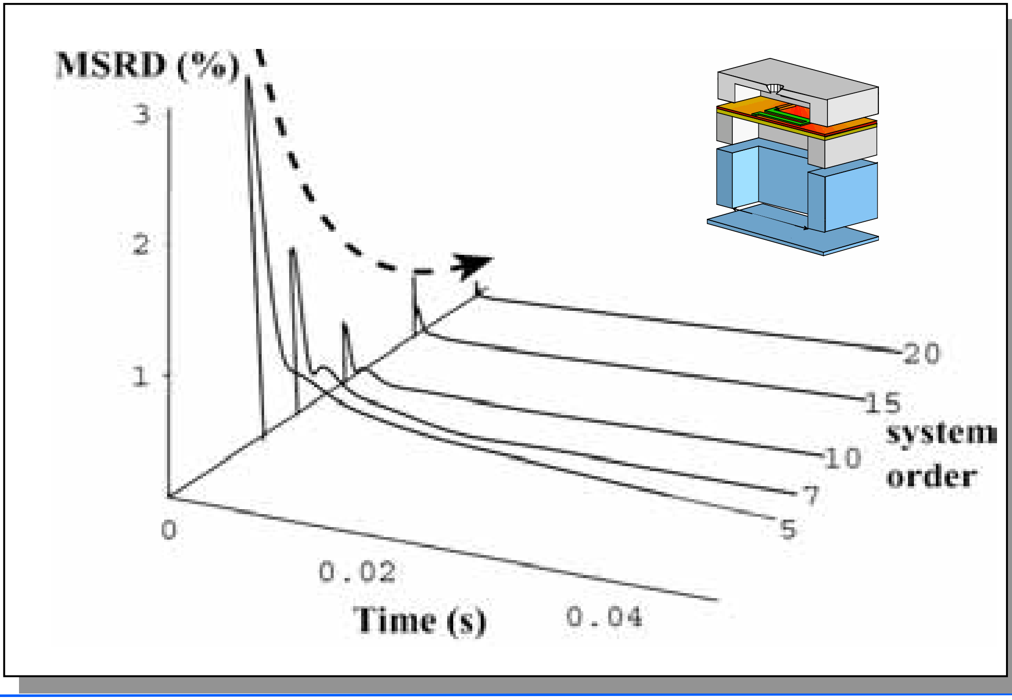
- If output array E is a unity matrix than

$$y_r = E^T \cdot V \cdot T = V \cdot z = \hat{T}_i$$

- E does not participate in Arnoldi, so each output is equally approximated

Mean Square Relative Difference

$$MSRD(t) = \sqrt{\frac{\sum_{i=1}^{n} \left(\frac{T_i(t) - \hat{T}_i(t)}{T_i(t)} \right)^2}{n}}$$



Nonlinear Input

$$R = R(T) = R_0 \cdot (1 + \alpha T + \beta T^2 + \ldots)$$

Nonlinear input

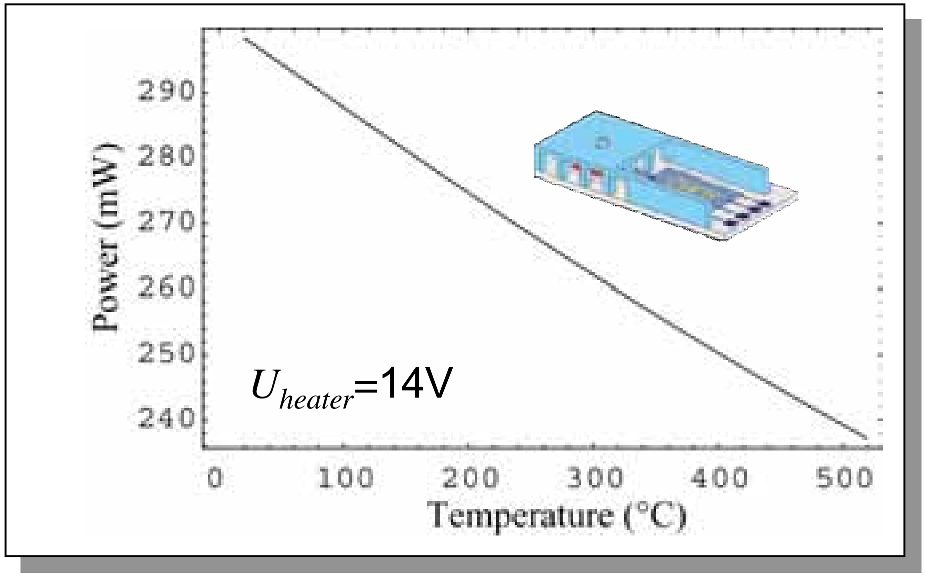
$$[C] \cdot \dot{T} + [K] \cdot T = F \cdot Q(t,T) = F \cdot \frac{U^2(t)}{R(T)}$$

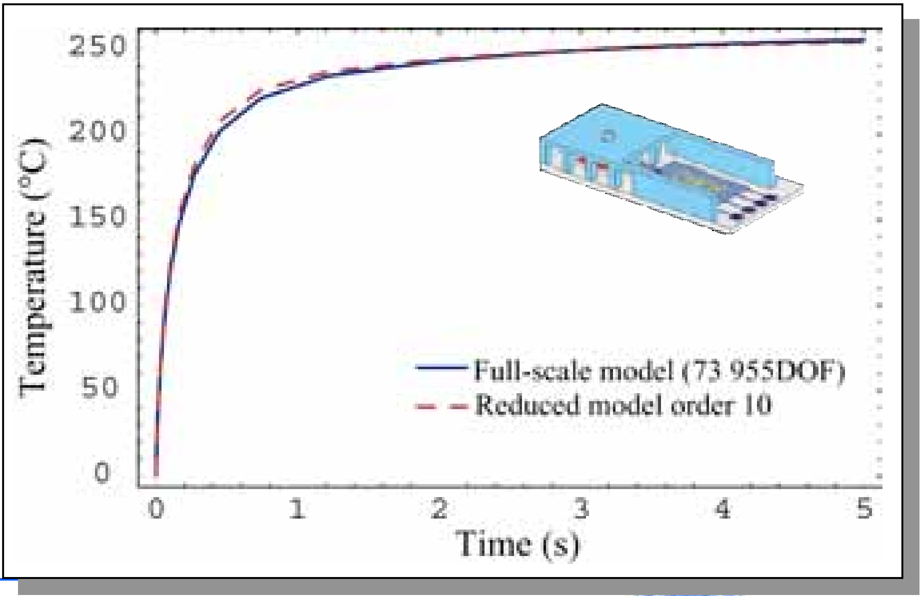
Input function does not participate in MOR

$$V^T CV \cdot \dot{z} + V^T KV \cdot z = V^T F \cdot Q(t, V^* \cdot z)$$

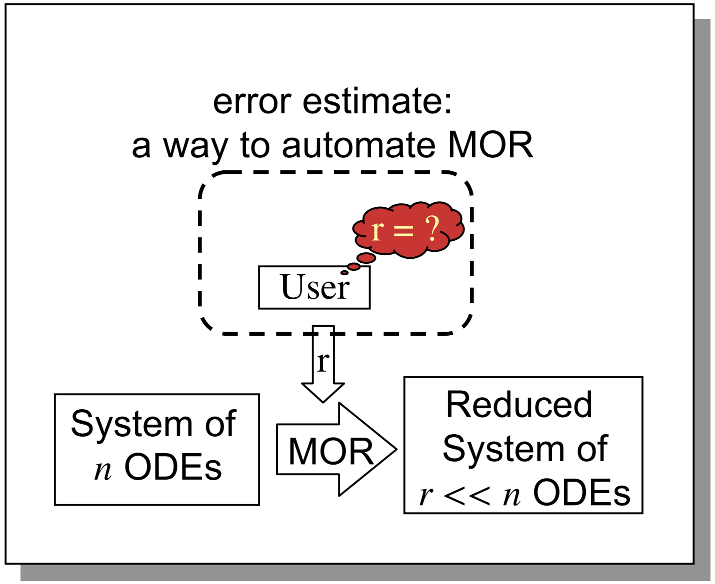
Control temperature is a single node temperature T_N

$$Q(t,T) = Q(T_N(t)) = Q(V^* \cdot z)$$





Error Indicators for Automatic MOR



- **Key question :**
  What is a suitable order of the reduced system for a desired accuracy?
- “Rule of thumb”: r = 50
- Proposed engineering approaches:
  - ☐ comparison of reduced systems of order r and $r + 1$
  - ☐ computation of Hankel singular values of the reduced system
  - ☐ sequential MOR

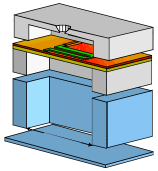
Convergence of Relative Error

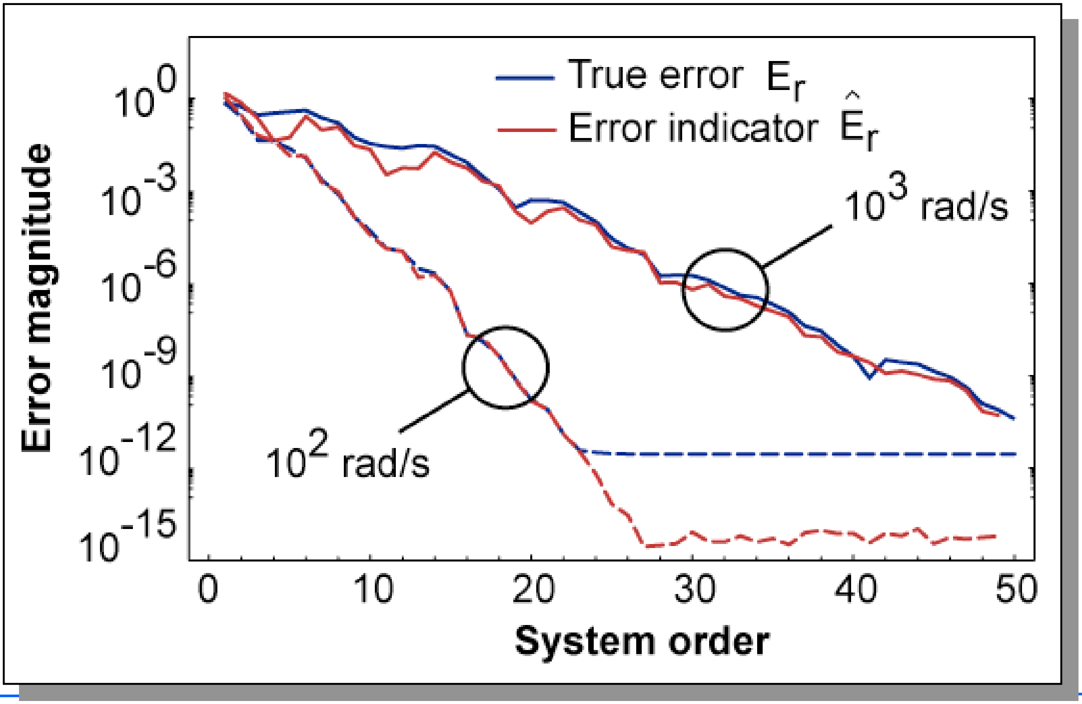
True error: $$E_r(s) = \frac{|G(s) - G_r(s)|}{|G(s)|}$$

Error indicator: $$\hat{E}_r(s) = \frac{|G_r(s) - G_{r+1}(s)|}{|G_r(s)|}$$

Main result:

$$\boxed{E_r(s) \approx \hat{E}_r(s)}$$





How to Choose an Optimal Dimension

- **Choose a maximum dimension for mor_for_ansys (-N)**
  - ✓A good starting point is from 30 (default) to 100.

- **Choose maximum frequency for monitoring error (application-dependent).**

- **Use LocalErrorIndicator to compute an error estimate as a function of a model dimension.**

- **A good idea is from time to time to compute a harmonic response in ANSYS and use LocalError to check whether LocalErrorIndicator is good enough.**

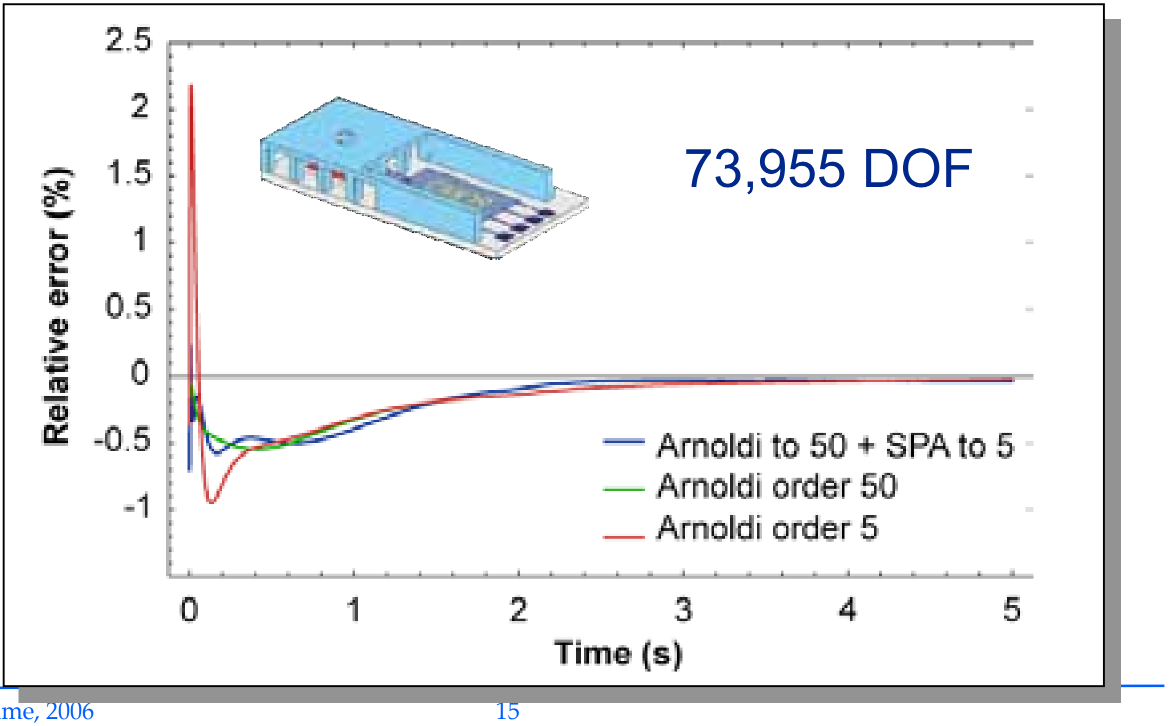
Sequential MOR

Use Arnoldi algorithm for reduction from n to r_1.

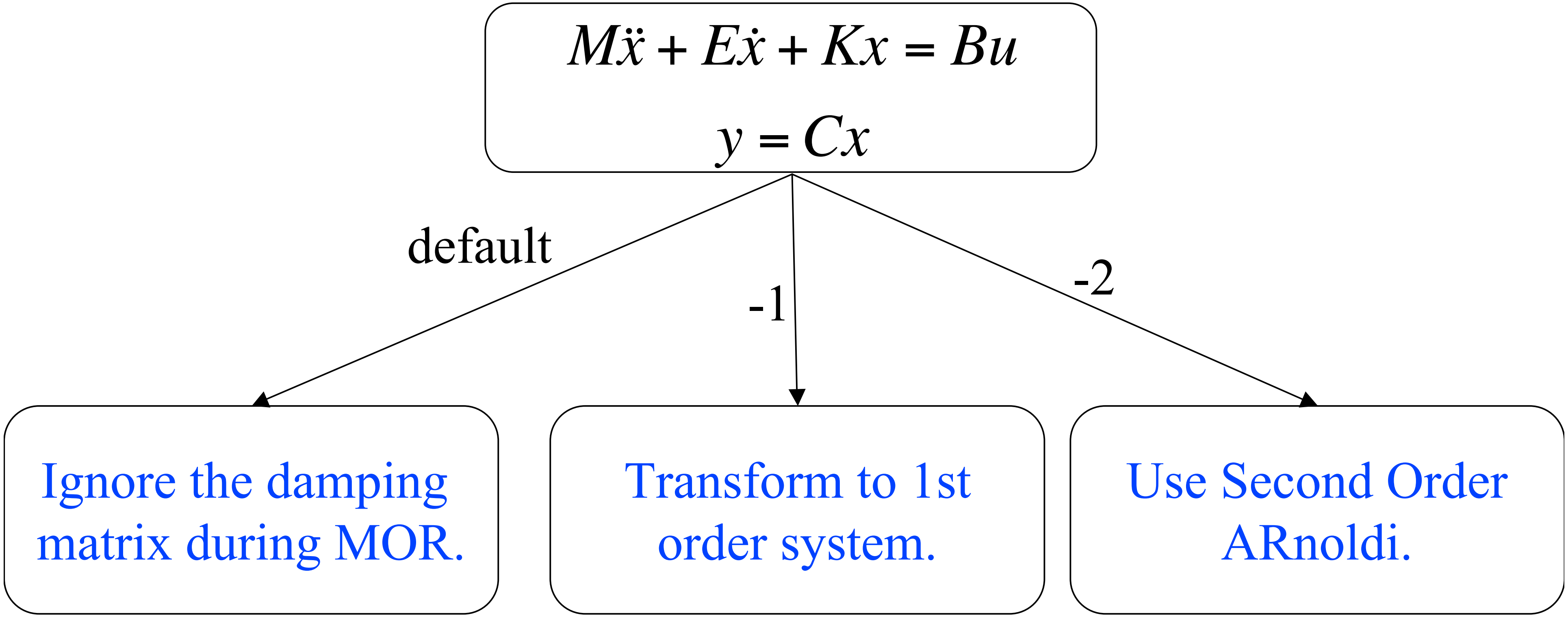
$$\left|G(s)\right| - \left|G_{r_2}(s)\right| \le \varepsilon_1 + \varepsilon_2$$

Control theory for further reduction from r_1 to r_2.

SPA (Singular Perturbation Approximation), $r_1 = 50$, $r_2 = 5$



Second Order Systems



$$M\ddot{x} + E\dot{x} + Kx = Bu$$
$$y = Cx$$

default

-1

-2

Ignore the damping matrix during MOR.

Transform to 1st order system.

Use Second Order ARnoldi.

- **The best in the case of proportional damping.**
- **Good for the general case but increases the problem dimension twice.**
- **Limited version: zero expansion point and one input.**

Proportional Damping

- Ignore the damping matrix during model reduction.

$$M\ddot{x} + E\dot{x} + Kx = Bu$$
$$y = Cx$$

$$\Im_{L,k}(K^{-1}M, K^{-1}B)$$

- Project the damping matrix afterwards.

$$E = \alpha M + \beta K$$

- The damping matrix can be obtained from the reduced matrices.

$$V^T E V = \alpha V^T M V + \beta V^T K V^T$$

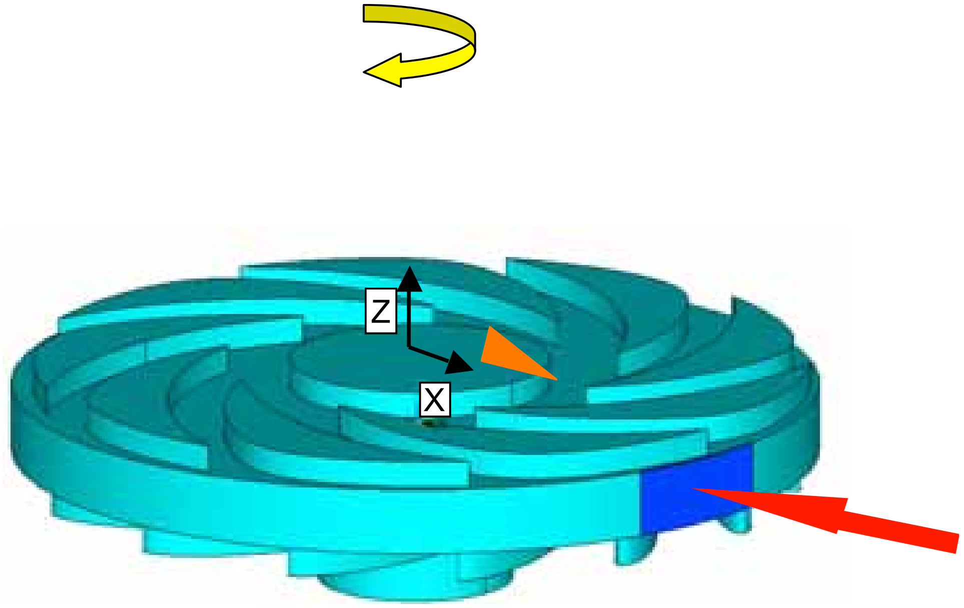
$$\hat{E} = \alpha \hat{M} + \beta \hat{K}$$

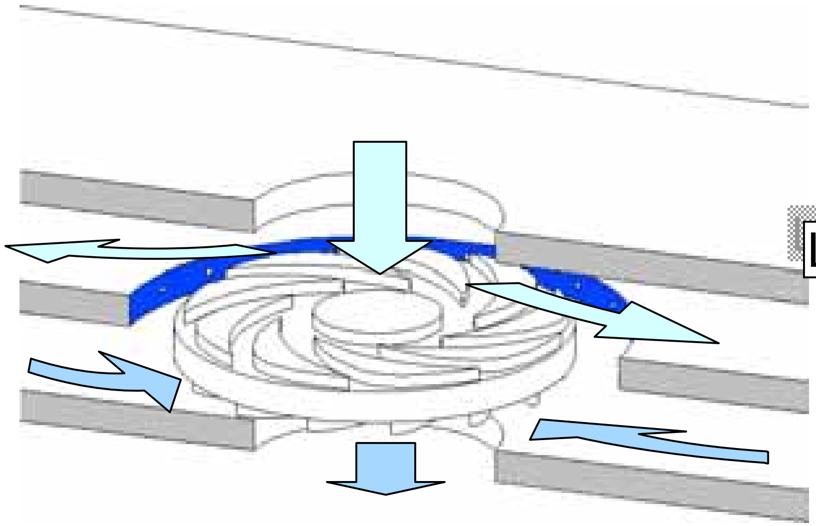
- You have alpha and beta as parameters.
- Moment are matched!

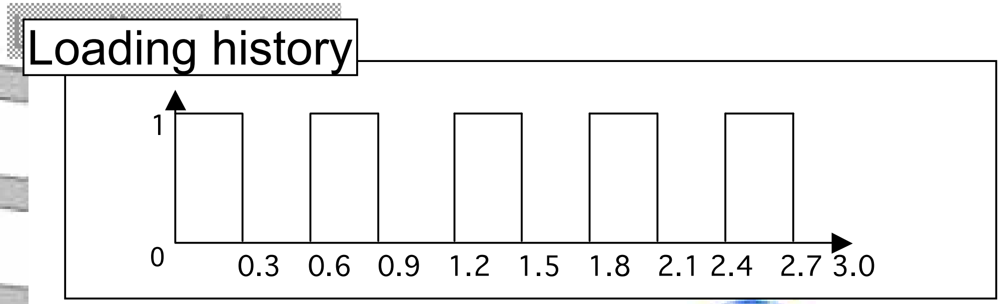
- Default in MOR for ANSYS.

Rotor Dynamics with MOR

- **Ritsumeikan University**
  - ✓Dr Asai, m-asai@se.ritsumei.ac.jp
  - ✓Prof Takano
  - ✓Prof Toriyama
- **Angular acceleration plus a load due to imperfection in design.**



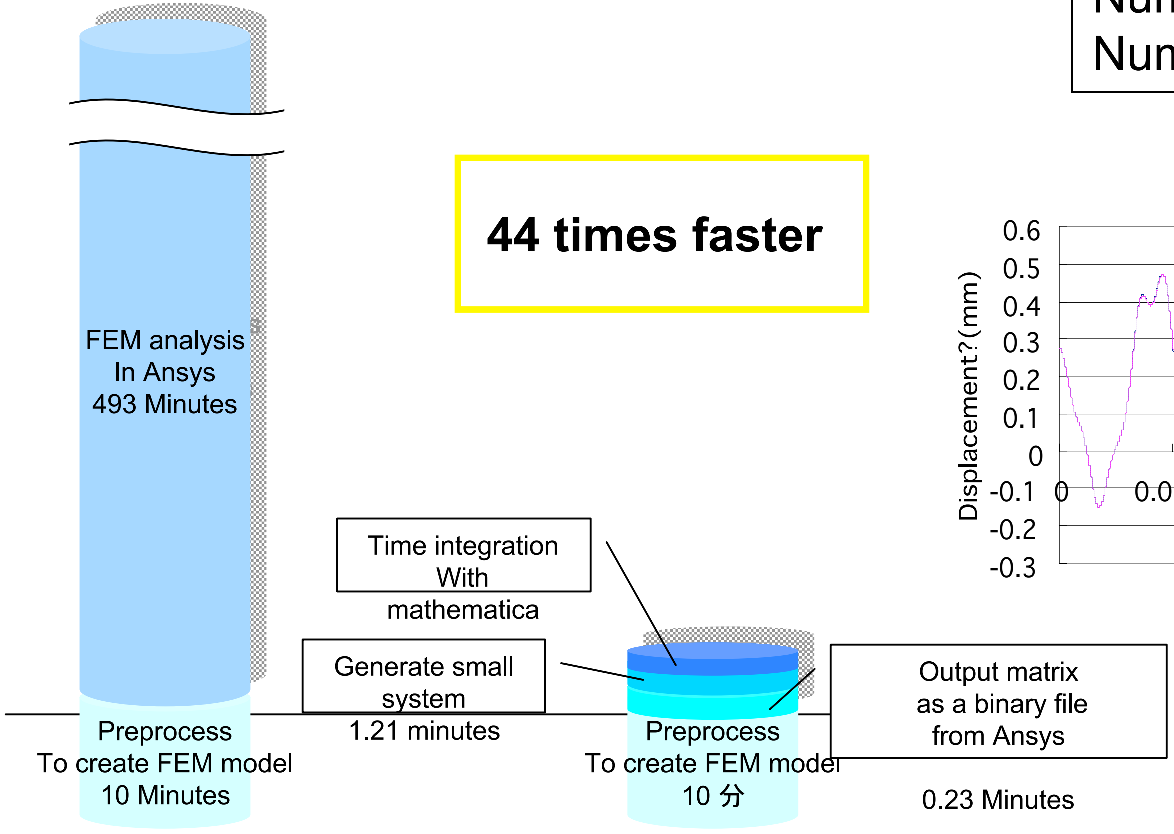




Comparison: ANSYS and mor4ansys

Number of elements 17653
Number of nodes 30106

44 times faster



FEM analysis
In Ansys
493 Minutes

Preprocess
To create FEM model
10 Minutes

Time integration
With
mathematica

Generate small
system
1.21 minutes

Preprocess
To create FEM model
10 分

Output matrix
as a binary file
from Ansys

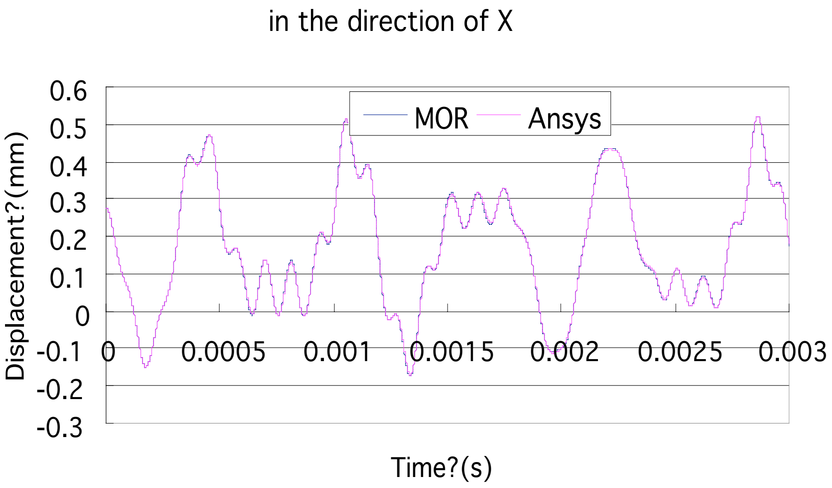
0.23 Minutes

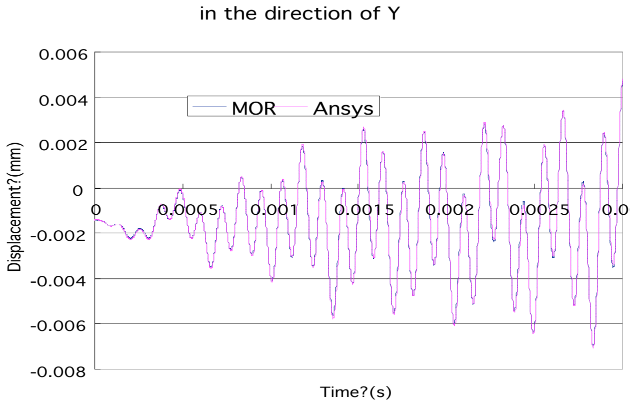
Total
503 minutes

Total
11.46 minutes

Ansys

MOR4Ansys





Optimization of a piezoresistive micro accelerometer

- **Prof Han, jshan@andong.ac.kr**

  - Formulation

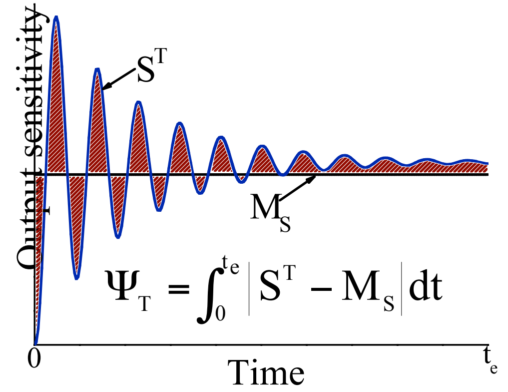
minimize $\Psi_T = \int_0^{t_e} \left| \boxed{S^T} - M_S \right| dt$

subject to $f_1 \geq f_{req}$

$\boxed{\delta^T_{50g}} \leq d_{gap}$

$\sigma_{2000g} \leq \sigma_{yield}$

transient information !



$S^T=0.5K\varepsilon V_a/a_y$ (output sensitivity)

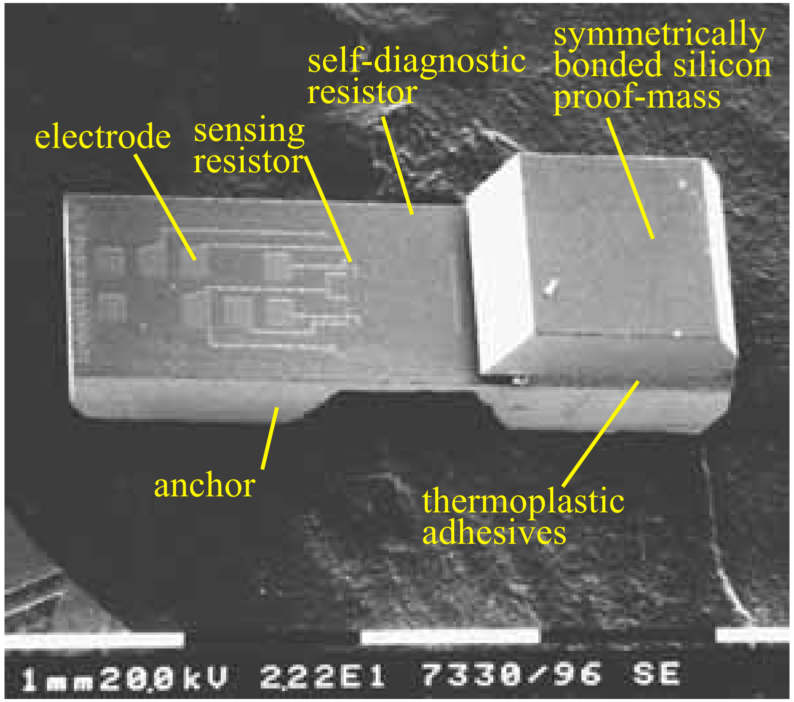
M_s=target value for S (=200 μV/g)

f_1=resonant frequency

δ^T_{50g}=max. transient disp. under 50 g

d_{gap}=disp. constraint (=3 μm)

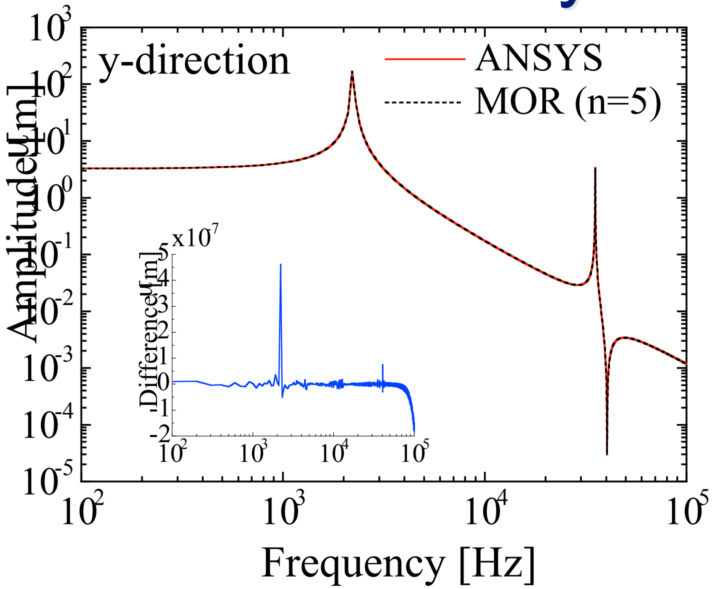
σ_{2000g}=von Mises stress under 2000 g



†: Ko J S, Cho Y H, Kwak B M, and Park K 1998 IMECE'98
Design and fabrication of piezoresistive cantilever microaccelerometer arrays with a symmetrically bonded proof-mass

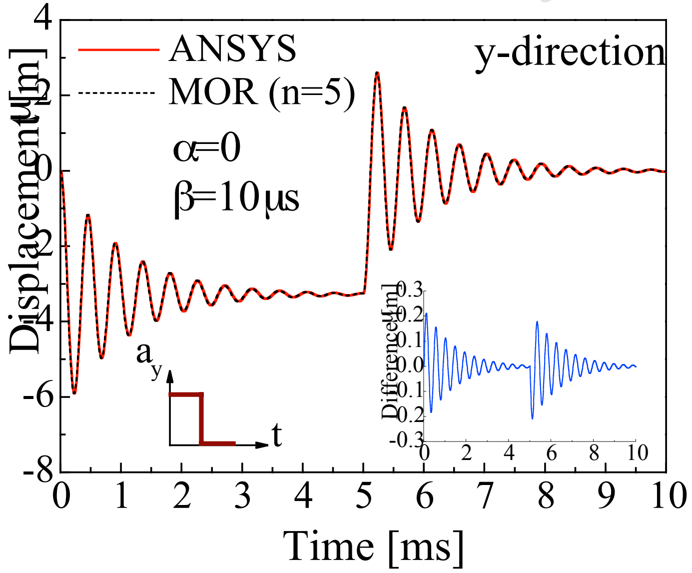
ANSYS vs. MOR

- Harmonic analysis



Time (in second)	Full ANSYS	MOR (n=5)
Total DoF	6 318	5
Time in ANSYS	11 618	60
Time in mor4ansys	-	6
Time in Mathematica	-	5
Total time	11 618	71

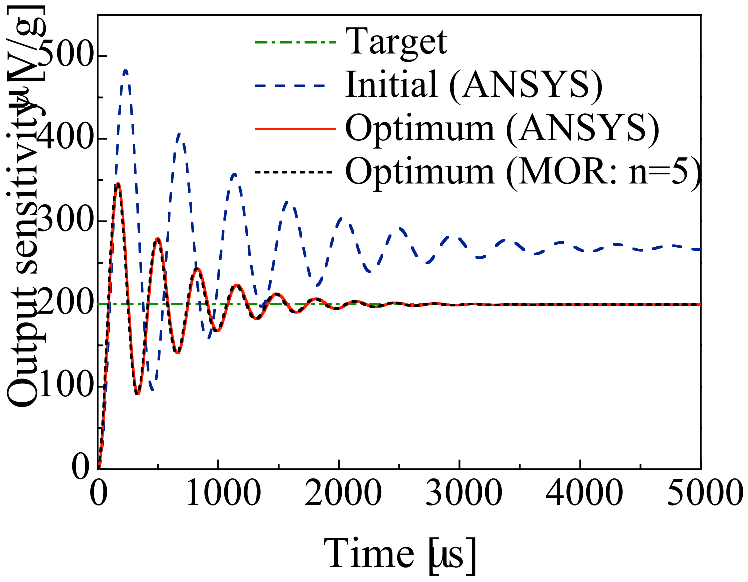
- Transient analysis



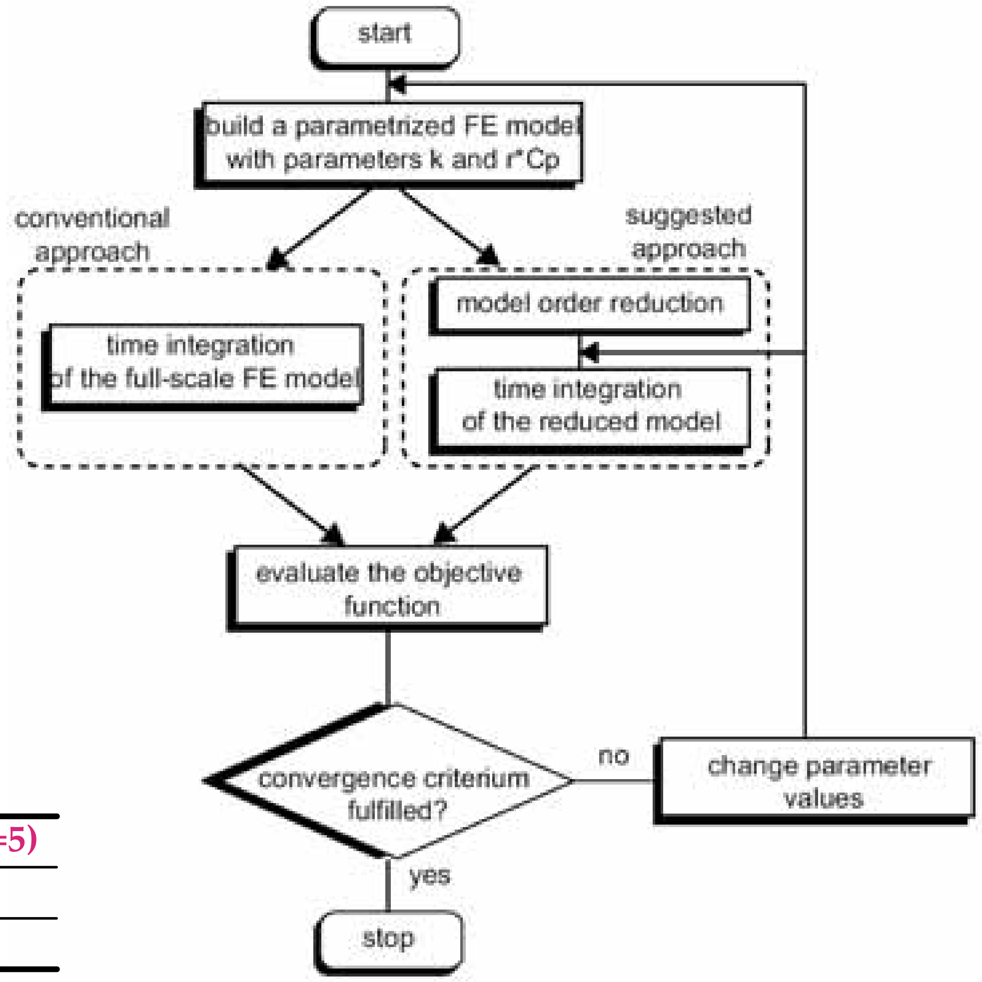
Time (in second)	Full ANSYS	MOR (n=5)
Total DoF	6 318	5
Time in ANSYS	4 000	60
Time in mor4ansys	-	5
Time in Mathematica	-	13
Total time	4 000	78

Optimization using MOR

- Transient sensitivity



Optimization	Full ANSYS	MOR (n=5)
No. of iterations	5	5
Total time in second	116 500	5 400



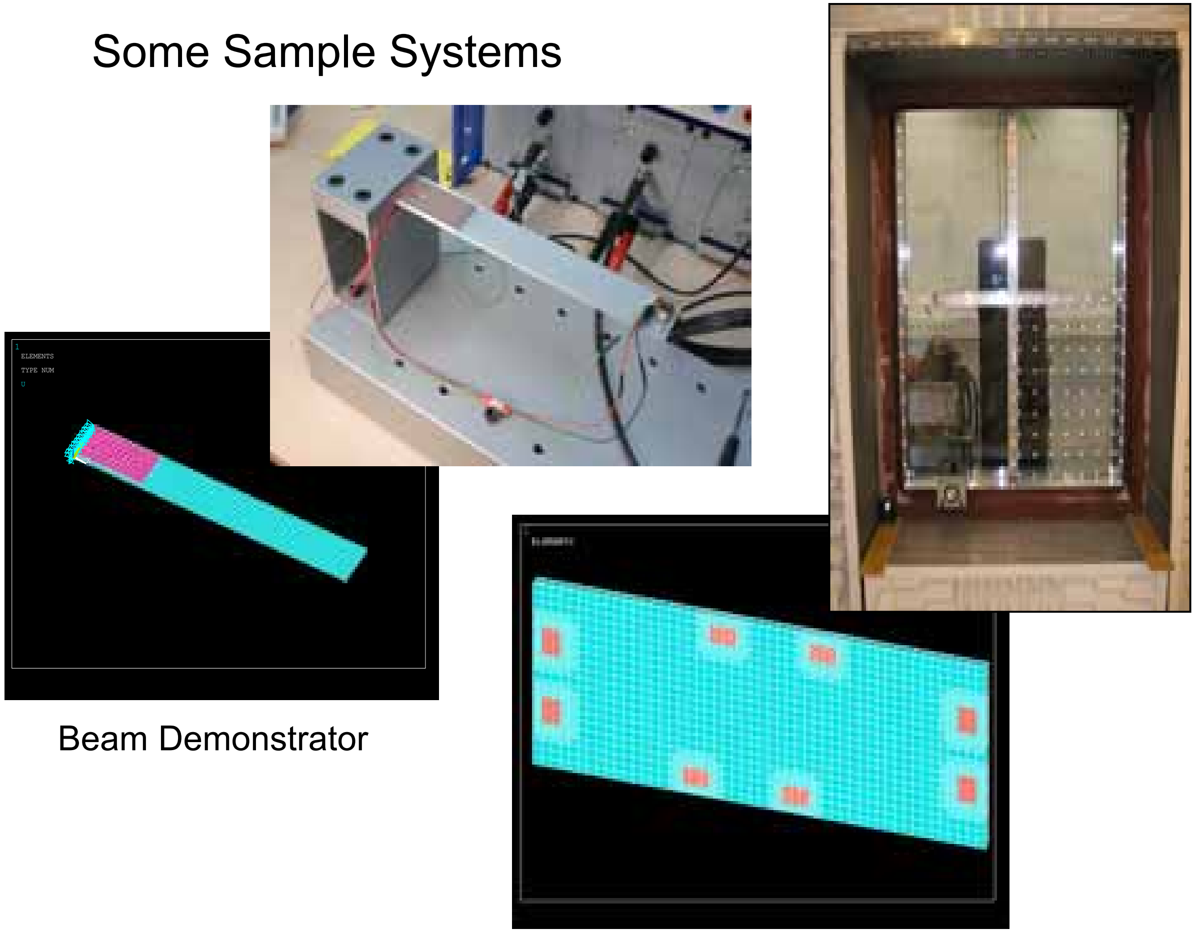
Piezoelectrics

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{V} \end{Bmatrix} + \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{V} \end{Bmatrix} + \begin{bmatrix} K & K^z \\ {K^z}^T & K^d \end{bmatrix} \begin{Bmatrix} u \\ V \end{Bmatrix} = \begin{Bmatrix} f \\ l \end{Bmatrix}$$

- **A fully coupled problem.**
- **Differential-algebraic equations.**
- **Stiffness matrix is indefinite.**
- **MOR for ANSYS 1.83**
  - ✓ $L^T DL$ solver is slow.
  - ✓ Use LU (but needs more memory).
- **Slides by Soong-Oh Han, han@szm.tu-darmstadt.de**

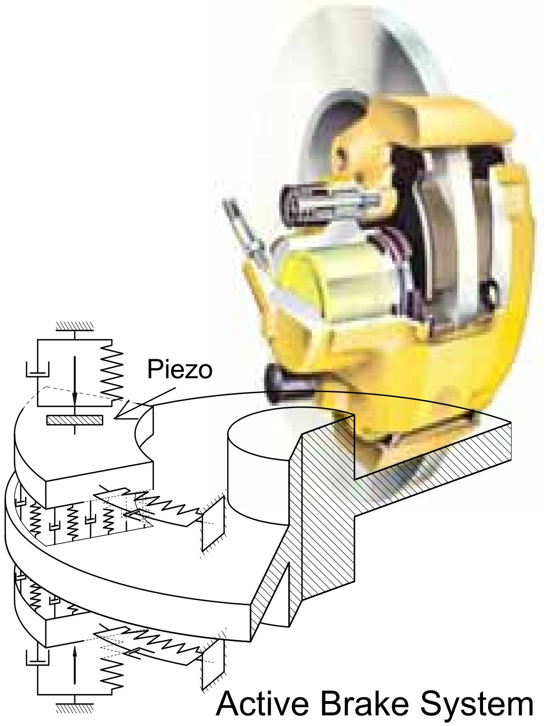
Modelling of Smart Structures with Piezoceramic Actuators

Some Sample Systems



Beam Demonstrator

Active Façade



Active Brake System

… Active Oilpan, Active Milling Machinery, etc.

Dipl.-Ing. Soong-Oh Han

Modelling of Smart Structures with Piezoceramic Actuators

Goal:

Modelling of complete active systems for vibration and noise reduction with implemented control to perform sensitvity and damage analyses

Main Problem:

FEM very suitable to determine dynamic behaviour of piezoceramic components and structures in general, but NOT capable of implementing control methods

Matlab/Simulink very suitable to implement and compare control algorithms but NOT capable of dealing with high order DOF models

Solution:

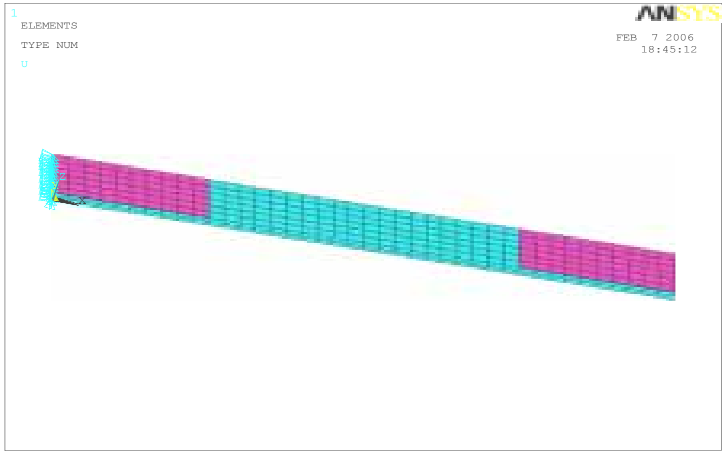
Obtaining a reduced model from the FEM model using MOR4Ansys and implementation in the Matlab environment

Advantages (compared to other reduction methods):

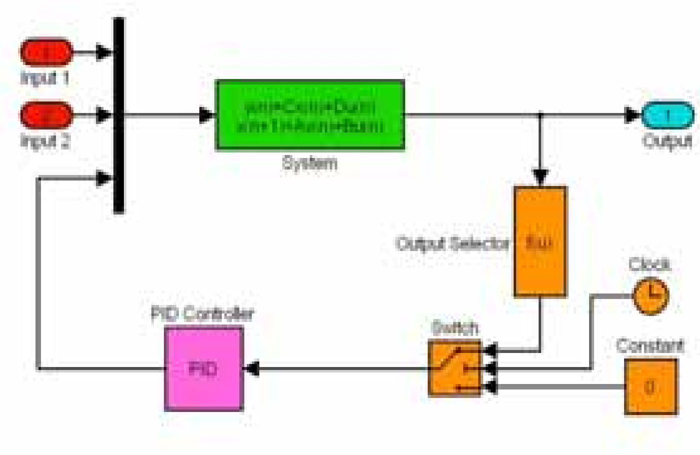
- Electric DOFs and In- /Outputs of the FEM model are maintained in the reduced system
- Automated computation of the reduced system possible, suitable for sensitivity analyses
- Very good performance of reduced system compared to the FEM model

Dipl.-Ing. Soong-Oh Han

Sample System: Beam Demonstrator



Reduction with MOR4Ansys

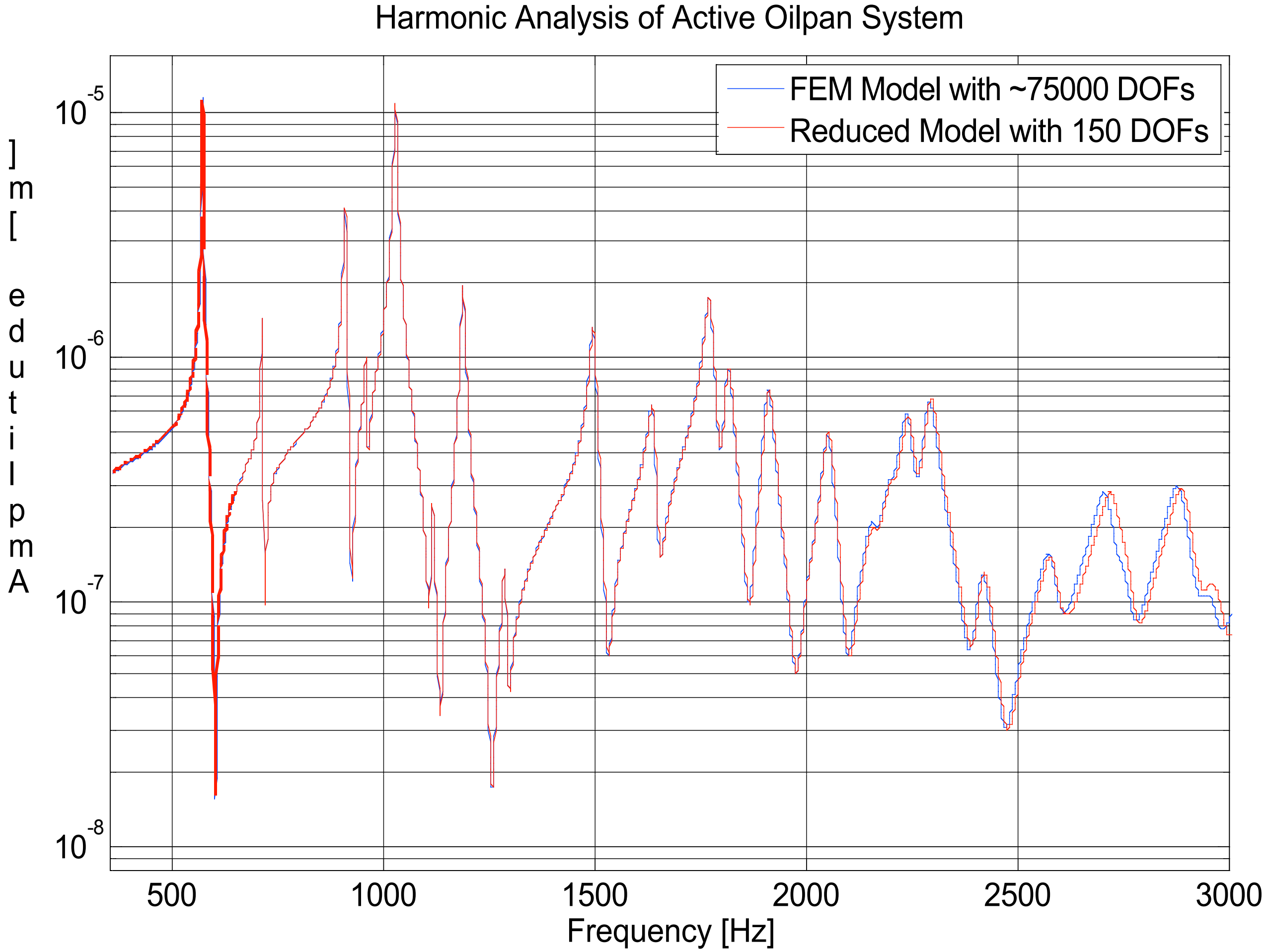


- Eigenfrequencies 67.103 Hz, 406.36 Hz, 1147.3 Hz, …
- 2982 DOFs

- Eigenfrequencies 66.134 Hz, 402.87 Hz, 1137.3 Hz, …
- 30 DOFs
- Input 1: Displacement at the clamping
- Input 2: Voltage applied on piezo at clamping
- Control Variable: Voltage applied on piezo at free end

Dipl.-Ing. Soong-Oh Han

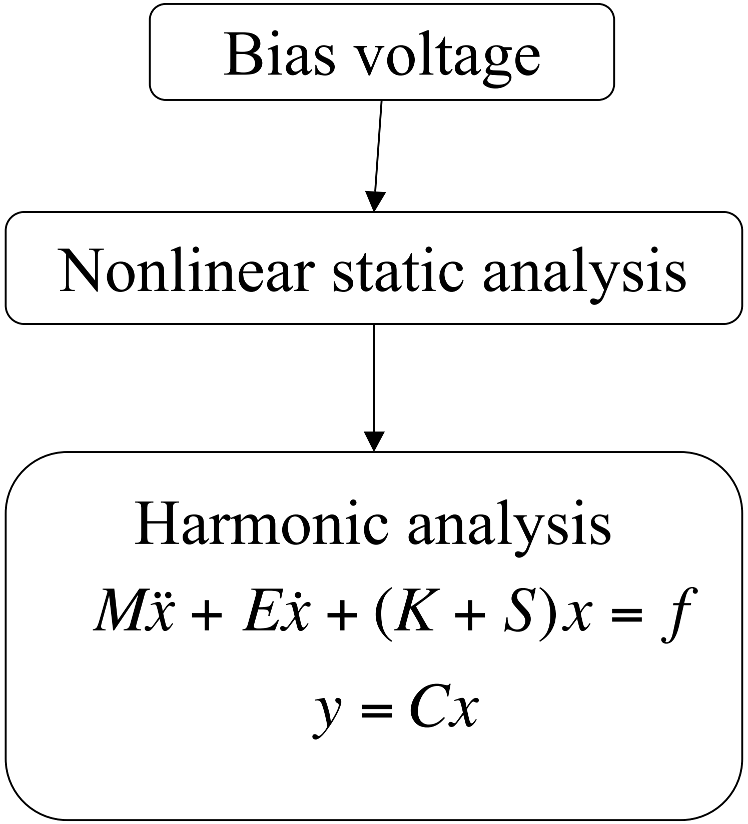
Sample System: Active Oilpan



Dipl.-Ing. Soong-Oh Han

Electromechanical System

- **TRANS126**
  - ✓capacitive response.
- **Nonlinear analysis.**
- **Pre-stressed harmonic analysis.**
- **Slides by Laura Del Tin, deltin@imtek.uni-freiburg.de**
- **Tutorial available at MOR for ANSYS site.**



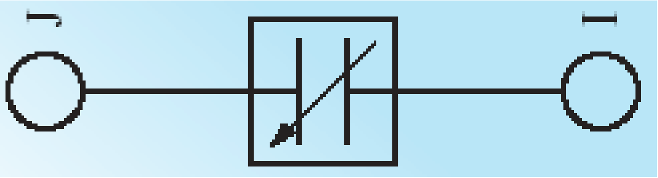
Bias voltage

Nonlinear static analysis

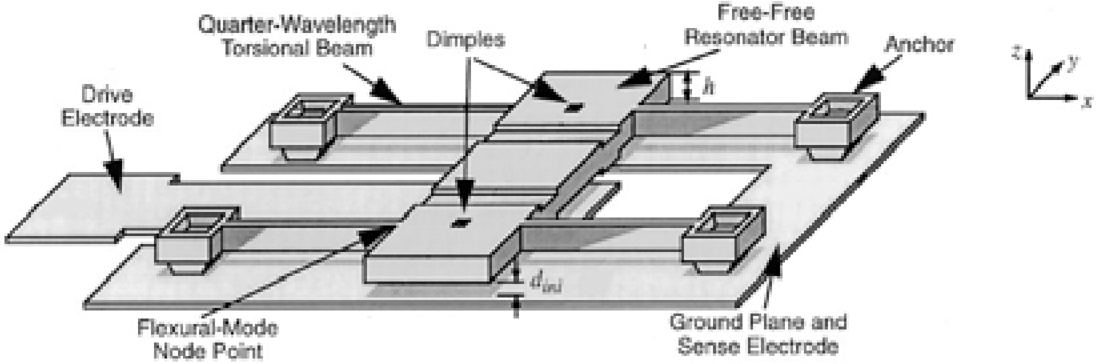
Harmonic analysis

$$M\ddot{x} + E\dot{x} + (K + S)x = f$$

$$y = Cx$$

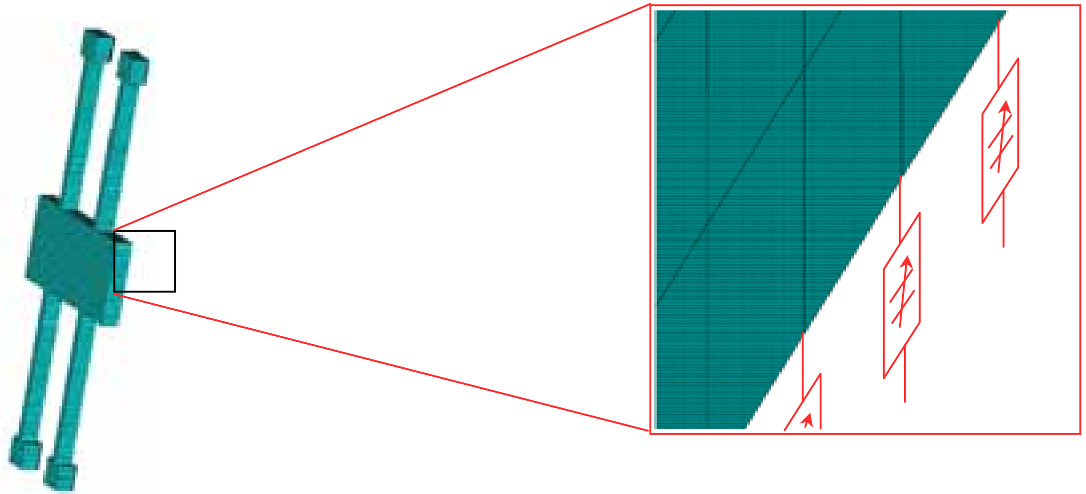


Case Study



Free-Free beam
one-port vertical
resonator

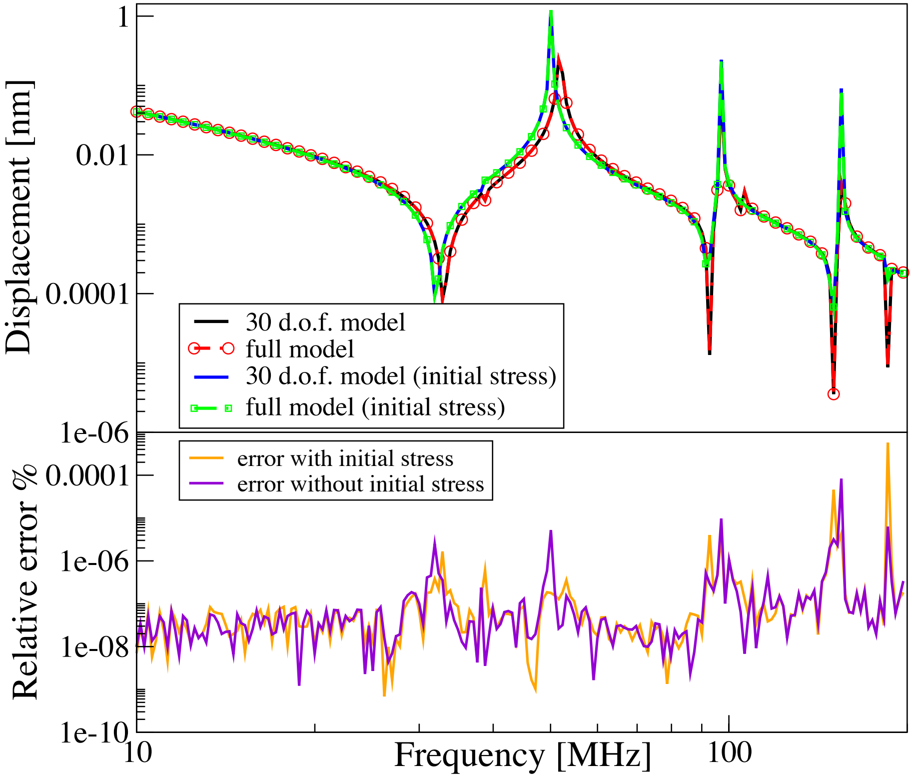
Wang, Nguyen, "VHF Free Free Beam High-Q Micromechanical Resonators", IEEE J. of Micromechanical system, vol. 9, n. 3, September 2000



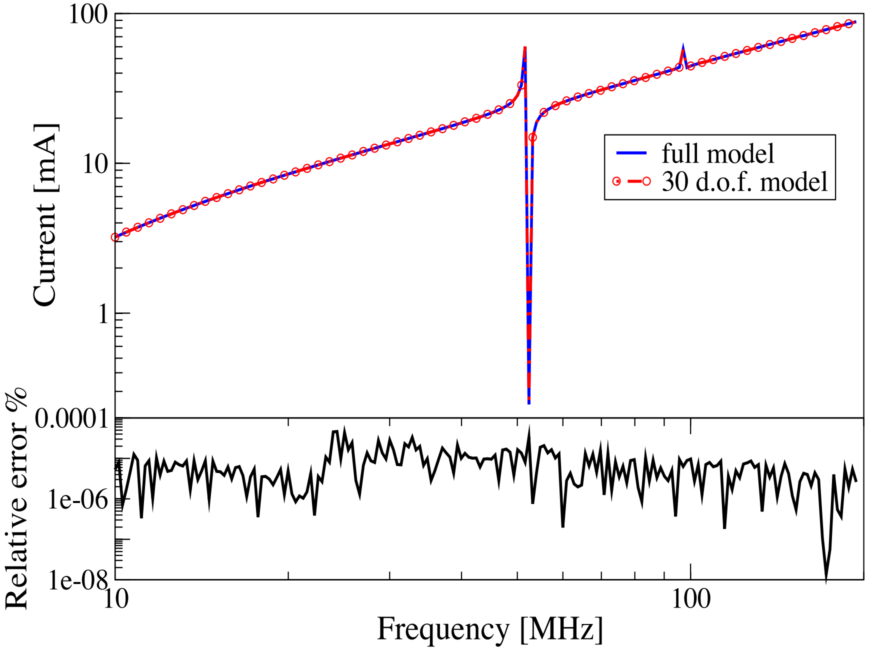
ANSYS FEM model with trans126 element

Results

Displacement amplitude central part of the beam, for no initial stress and 100MPa initial stress

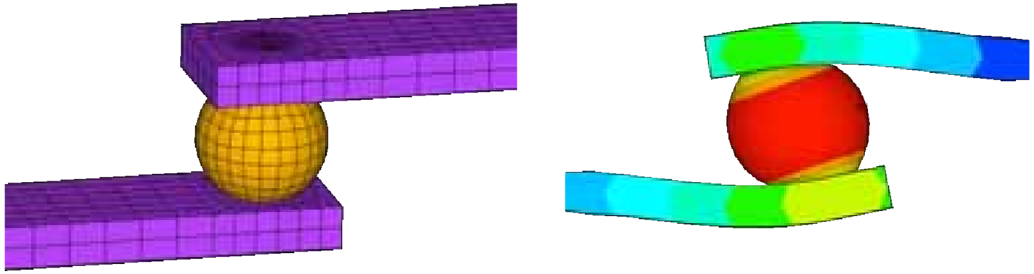


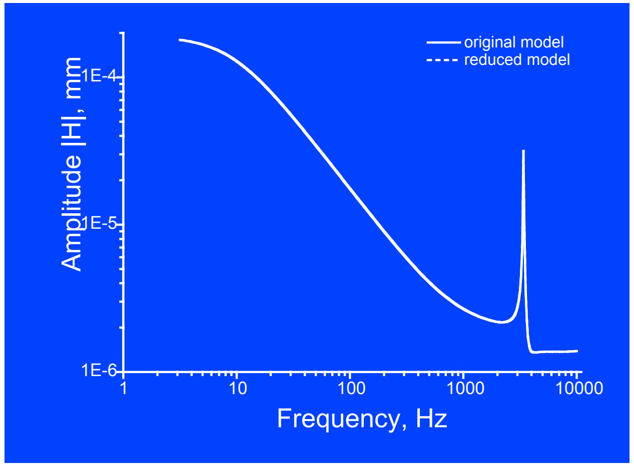
Short-circuit output current, for an applied bias voltage Vb = 86 V and a small-signal voltage of 1 volt

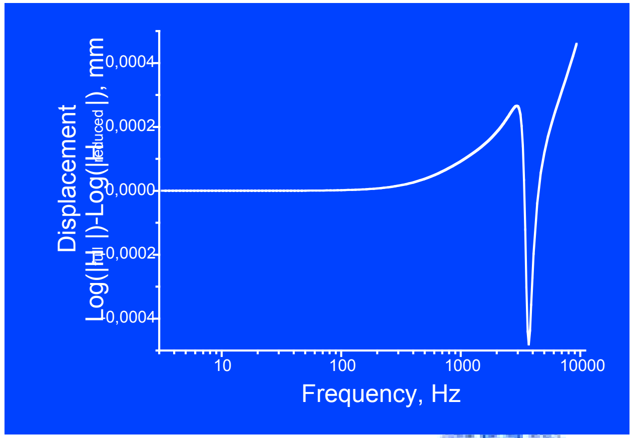


Thermomechanical Problem

$$\begin{bmatrix} M_U & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{T} \end{Bmatrix} + \begin{bmatrix} E_U & 0 \\ 0 & E_T \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{T} \end{Bmatrix} + \begin{bmatrix} K_U & K_{UT} \\ 0 & K_T \end{bmatrix} \begin{Bmatrix} u \\ T \end{Bmatrix} = \begin{Bmatrix} f \\ Q \end{Bmatrix}$$







- **Non-uniform temperature.**
- **Matrix K_{UT} is not available but can be re-constructed.**
- **One can treat mechanical part as quasi-stationary, $M_U=0$, $E_U=0$.**

Krylov Subspace Techniques for Coupled Structural Acoustic Analysis

R Srinivasan Puri, Dr. Denise Morrey

Oxford Brookes University,

Vehicle Engineering Group,

School of Technology,

Headington , Oxford OX3 0BD

United Kingdom.

Problem Description

Compute pressure level at drivers ear location (automobile or an aircraft interior) under structural or acoustic excitation.

Classical fully coupled FSI Formulation:

Structure Matrices

Displacements

$$\begin{pmatrix} Ms & 0 \\ Mfs & Ma \end{pmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix} + \begin{pmatrix} Cs & 0 \\ 0 & Ca \end{pmatrix} \begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix} + \begin{pmatrix} Ks & Kfs \\ 0 & Ka \end{pmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} Fs \\ 0 \end{Bmatrix}$$

Coupling term

Fluid Matrix

Pressures

- The Direct formulation cannot be avoided in most cases especially if spatial damping treatment is present.
- Unsymmetric Mass, Stiffness Matrix increases computational expense.
- Modelling the final trim parts and joints leads to very high mesh density, and results in huge computational time.

Test Case: 1

Clamped undamped Aluminium plate of 1m x 1m backed by a rigid walled cavity of dimensions 1m x 1m x 1m.

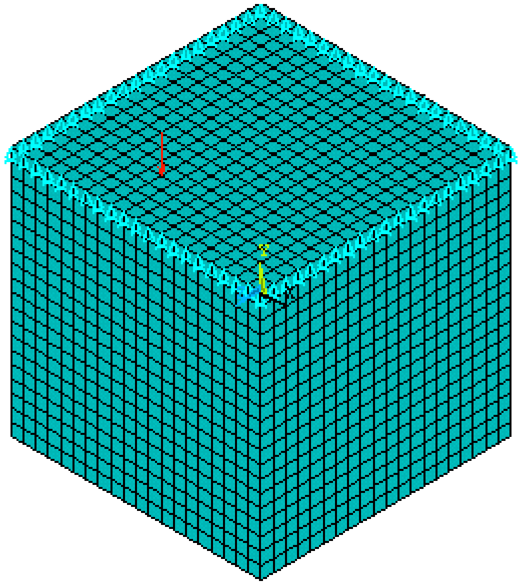


Fig:2: Coupled plate/cavity FE Model

- Point source excitation on structural node of the coupled model
- Compute resultant noise transfer function (P/F) at certain points in the fluid domain.

Test Case: 2

Test structure made of a combination of beams and plates.

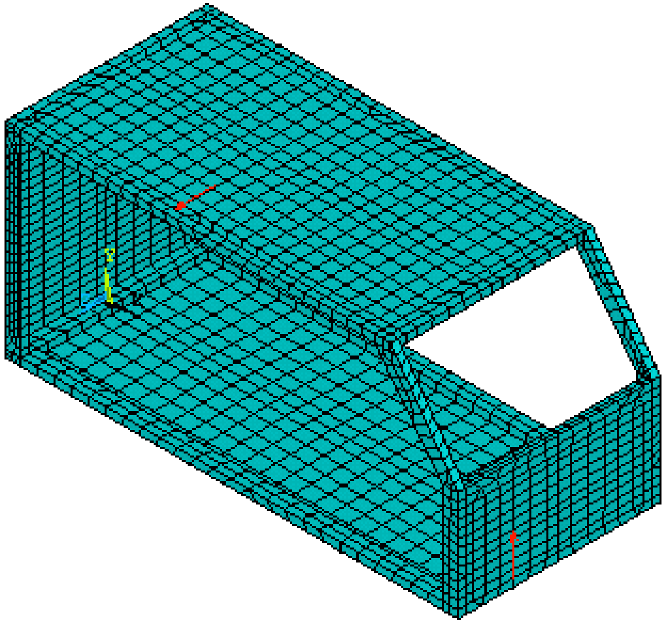


Fig:2: Structural FE Model

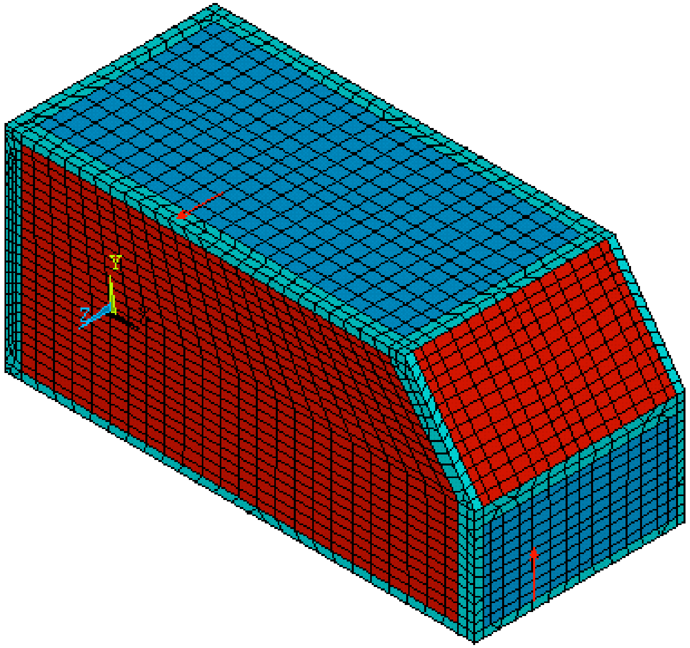


Fig:2: Coupled FE Model

- 2 Point sources - global Y, Z excitation - on structural nodes of the structural portion of the coupled model
- Compute resultant noise transfer function (P/F) at certain points in the fluid domain.

Results: Accuracy of Projection Framework

Test Case: 1

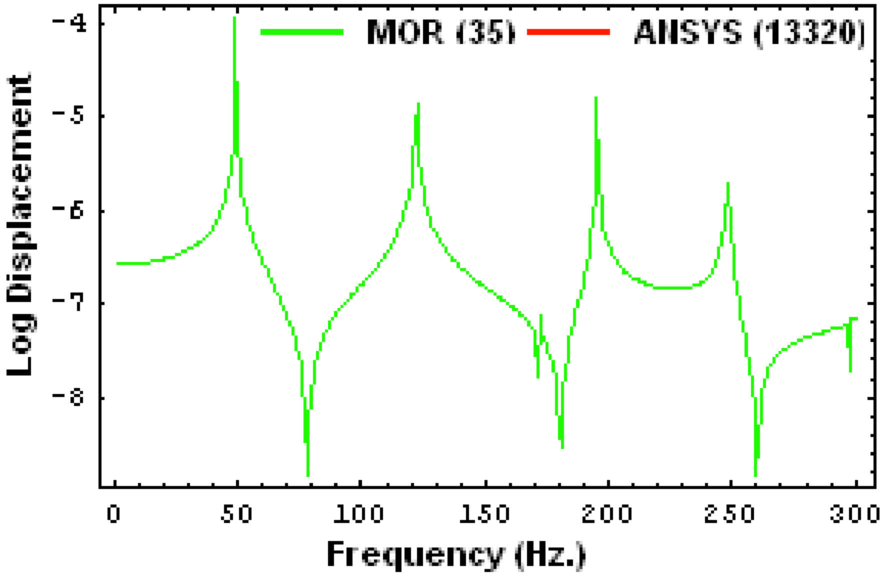


Fig:2: Displacement transfer function (Receptence)

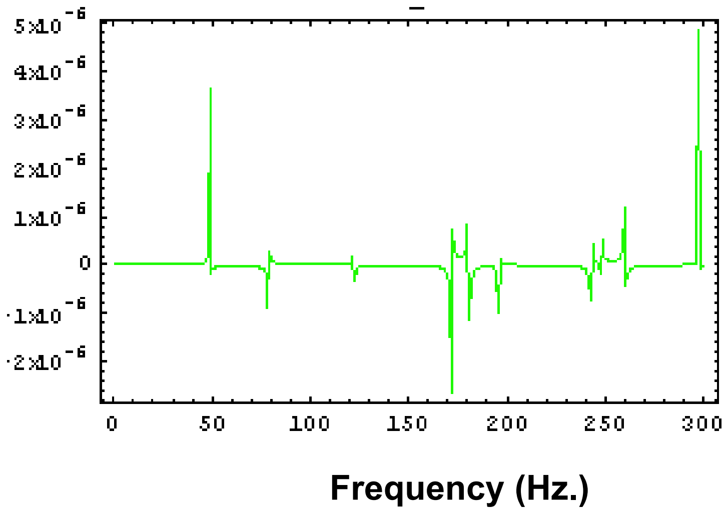


Fig:3: Error Plot

Order of higher dimensional system: 13,320

Order of reduced order model : 35 (Optimum)

Results: Accuracy of projection framework

Test Case: 1 (Contd..)

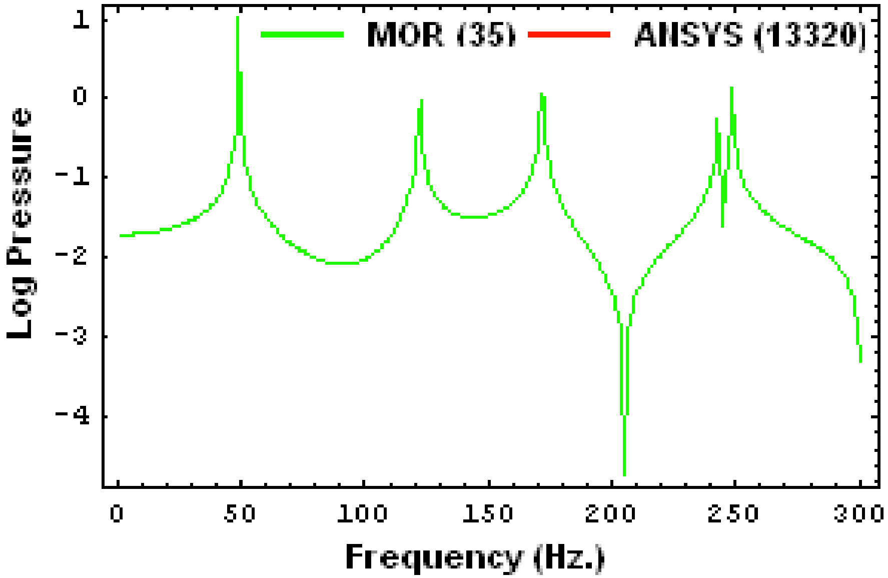


Fig:4: Noise transfer function (P/F)

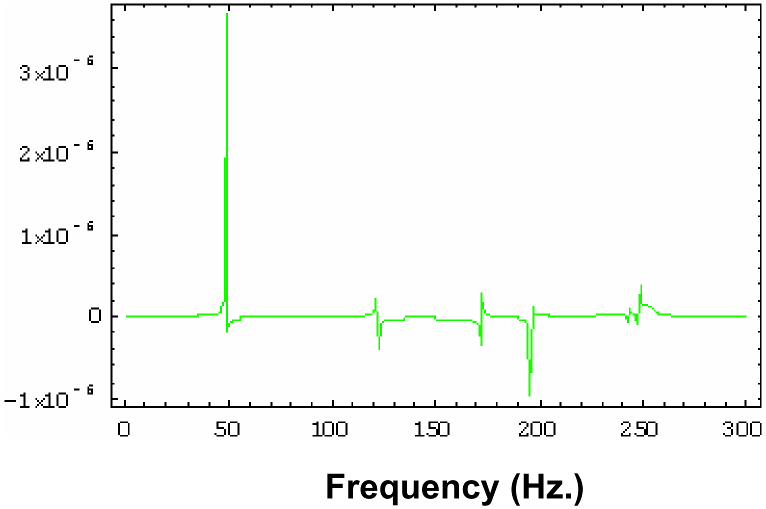


Fig:5: Error Plot

Order of higher dimensional system: 13,320

Order of reduced order model : 35 (Optimum)

Results: Accuracy of projection framework

Test Case: 2

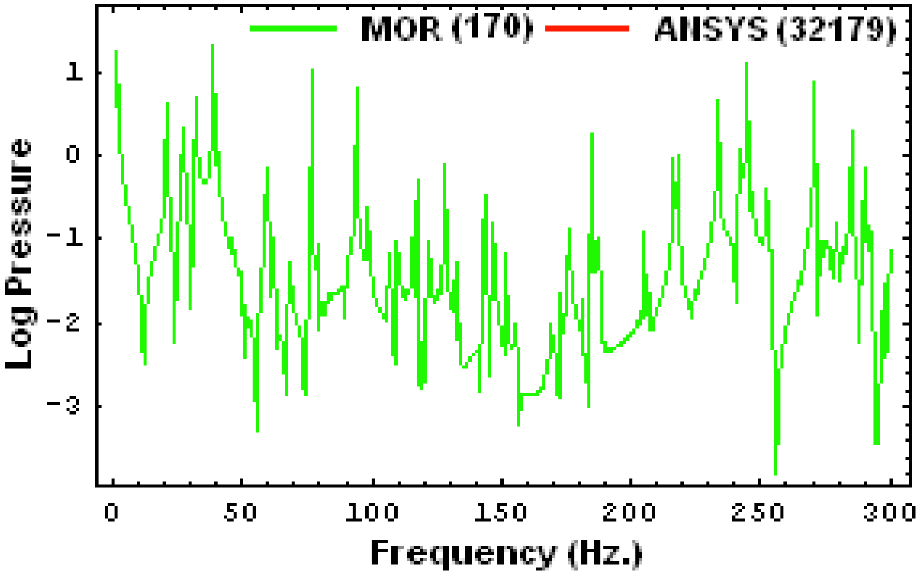


Fig:6: Noise transfer function (P/F)

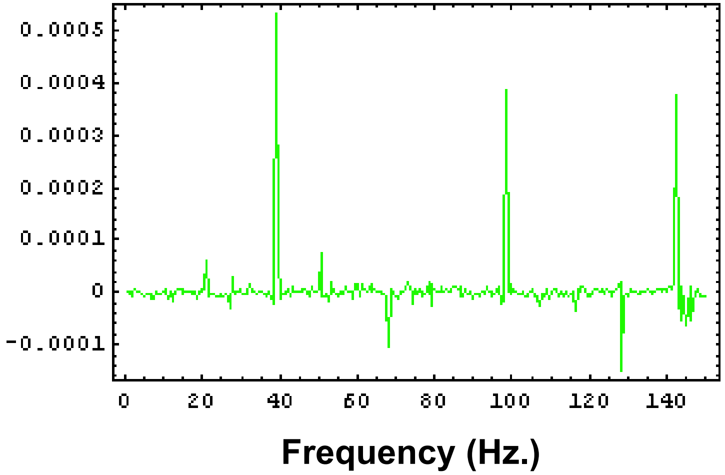


Fig:7: Error Plot

Order of higher dimensional system: 32,179

Order of reduced order model : 170 (Optimum)

Results: Computational Times

Model	Elements	DOF's	ANSYS Direct	MOR via Arnoldi	Reduction
TC^1	**8400**	**11427**	**2906 s**	**27.8 s**	**99.04 %**
TC^2	**14220**	**29413**	**16530 s**	**169.4 s**	**98.97 %**

Table 1 – Computational Times; TC1: Test Case-1; TC2: Test Case-2.

Model	ANSYS Stationary	Read Matrices , Arnoldi Vector Generation	Projection of Matrices	Reduced model Simulation	Total: MOR via Arnoldi
TC^1	**6 s**	**21.3 s (35 Vectors)**	**0.4 s**	**0.2 s**	**27.8 s**
TC^2	**4 s**	**144.7 s (170 Vectors)**	**14.7 s**	**6 s**	**169.4 s**

Table 2 – MOR Split Computational Times; TC1: Test Case-1; TC2: Test Case-2.

Conclusion

- Electro-thermal MEMS
- Structural mechanics
- Piezoelectric actuators for control
- Pre-stressed small-signal analysis for RF-MEMS
- Thermomechanical models
- Acoustics including fluid-structure interactions