Compact Electro-Thermal Model for a Microthruster: A Case Study

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FW5 EU MicroPyros project

- Developing a microthruster array for nanosatellites.

Main Site

- www.laas.fr/Micropyros/

Our Preprints

- www.imtek.de/simulation/

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- Discussions with all our partners.
- Tamara Bechtold, PhD student.

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- Microthruster Array
- Simulation Strategy
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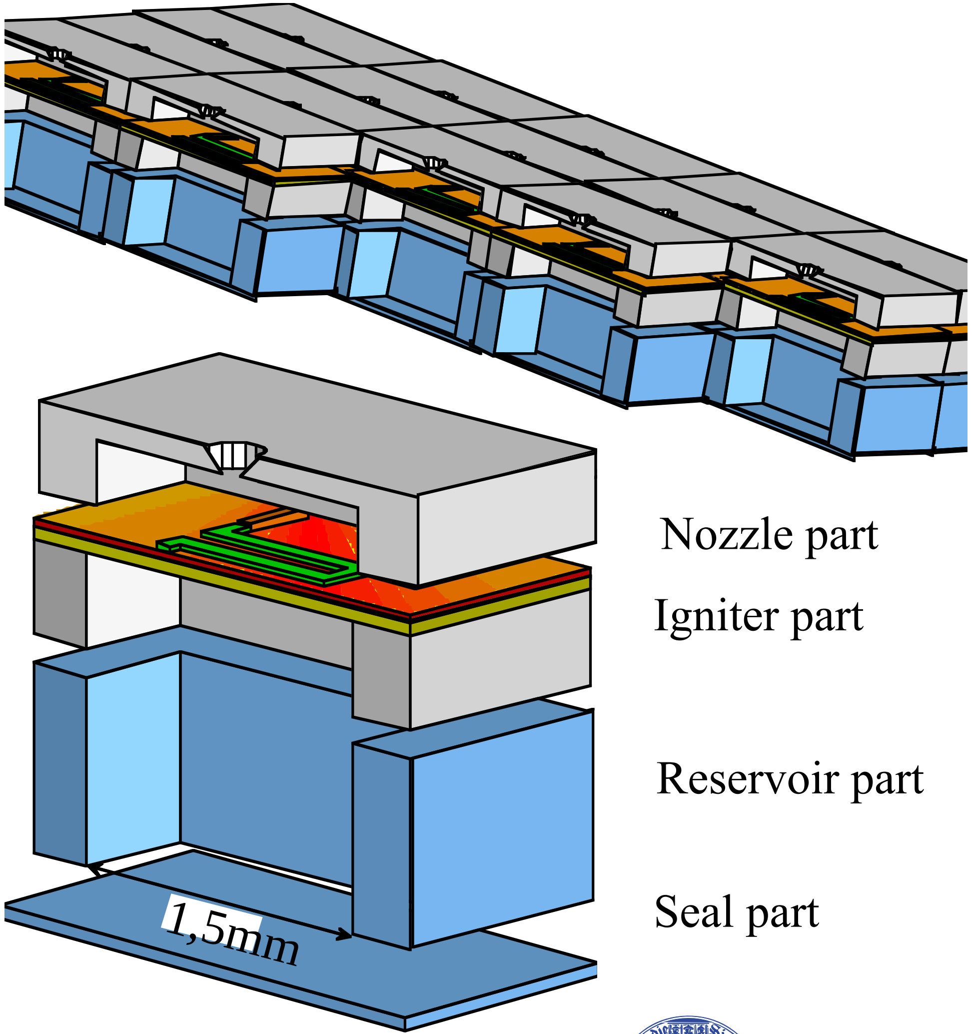
- Numerical Results

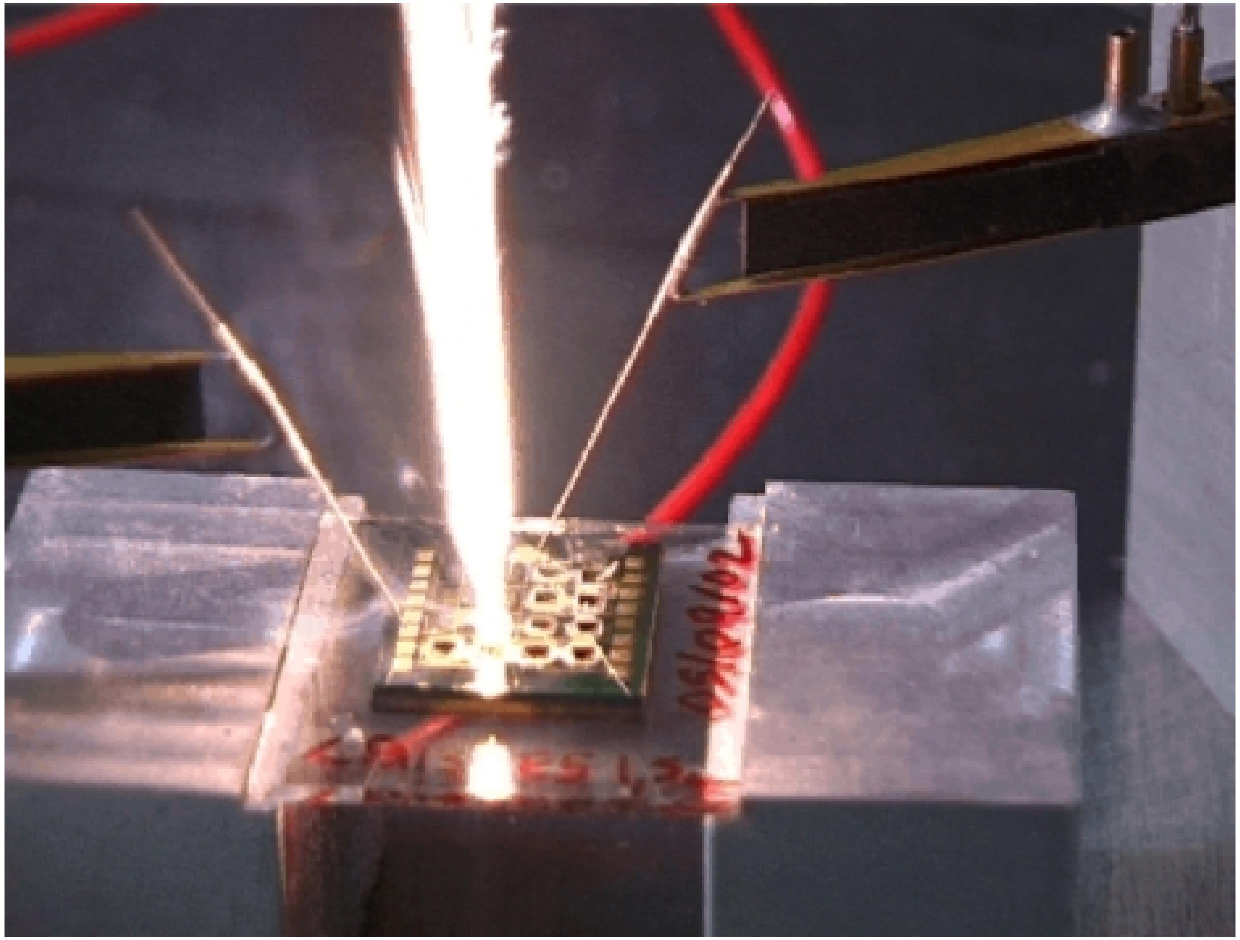
Microthruster Array

Overview

- A thruster is the crucial part of a nanosatellite.
- Solid propellant microthruster array:
  - High-energetic solid fuel,
  - No moving parts,
  - Manufactured from three wafers.
- Disadvantage: lack of restart ability.
  - May be compensated by manufacturing an array.
  - 10x10 thrusters on a chip.



4x4 Demo



Simulation Strategy

Key Engineering Questions

- To choose:
  - a wafer material (silicon, ceramics, or glass) and a technology to manufacture it,
  - a technology for the low-temperature bonding of the wafers,
  - a solid fuel composition and a technology for its filling into mm square cavities,
  - a packaging technology.
- To develop:
  - the optimal geometrical design for the microthruster array,
  - an intelligent driving circuit to operate it.

Simulation Strategy

Microthruster Theory of Operation

- Electro-Thermal Process
- Ignition and Sustained Combustion
- Membrane Rupture
- Gas Dynamics

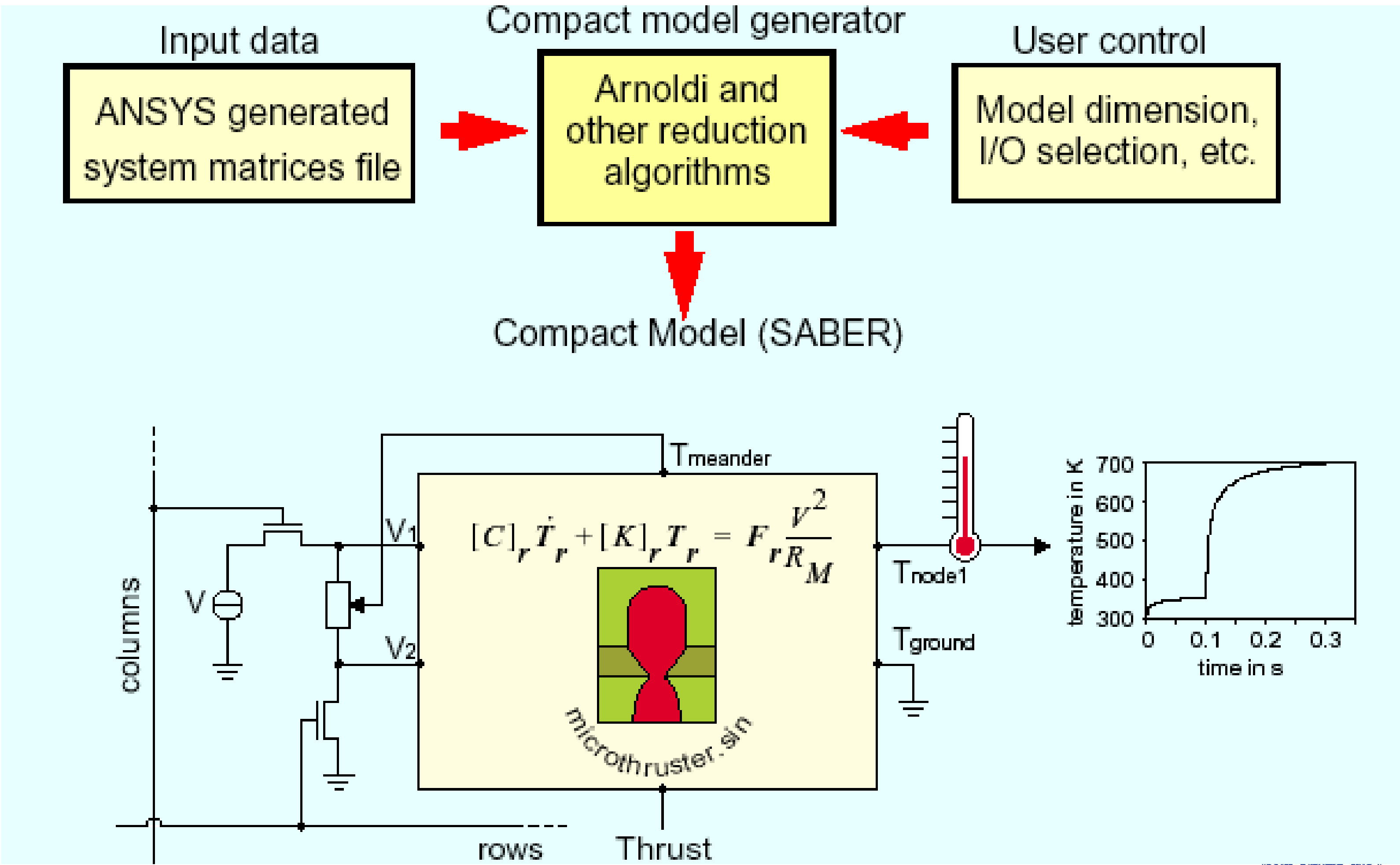
Main points

- Gas dynamics part should be simple:
  - Possibilities to optimize the nozzle are limited.
- Key question: the array integration density:
  - How close can be microthrusters to each other.
  - Heat management is very important.
  - What is the smallest width of the microthruster.
- Model order reduction:
  - Key problem in electro-thermal simulation.

Software to develop

Name	*Short description*	*Type*
EleThermo	Simulating electro-thermal ignition	ANSYS script
FilmCoef	Estimating film coefficient	Mathematica notebook
HeatTran	Simulating heat transfer during sustained combustion	ANSYS script
Thrust	Estimating impulse produced by the microthruster	Mathematica notebook
CoGen	Performing model order reduction	C++

Model Reduction for System Level Simulation



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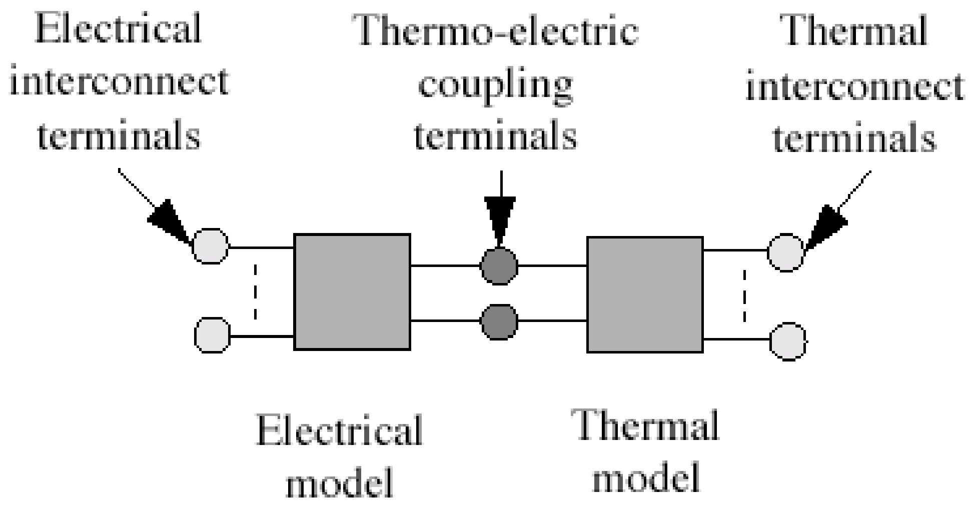
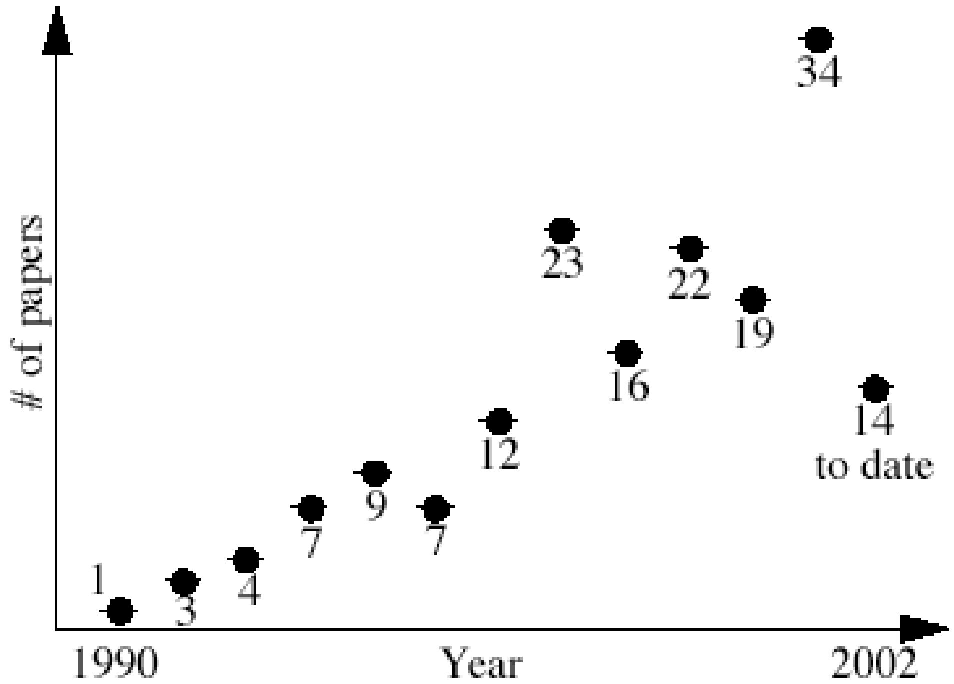
Electro-Thermal Simulation

Goal

- To treat electrical and thermal model as black boxes:



- Quite an important problem: number of IEEE papers on electro-thermal simulations



Electro-Thermal Process

- Heat generation by means of electrical current

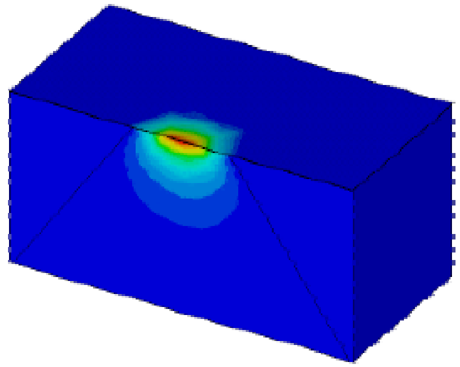
$$\nabla \bullet \boldsymbol{j} = 0, \boldsymbol{j} = \sigma \nabla \psi, Q = \frac{\boldsymbol{j}^2}{\sigma}$$

- Heat transfer

$$\nabla \bullet (\kappa \nabla T) + Q - \rho C_p \frac{\partial T}{\partial t} = 0$$

- Lumped resistor

$Q = I^2 R / V$ (V - resistor volume)



- Coupled Maxwell and heat transfer equations.
- Coupled Poisson and heat transfer equations.
- Lumped resistor and continuum-based heat transfer.
- Lumped heat transfer.

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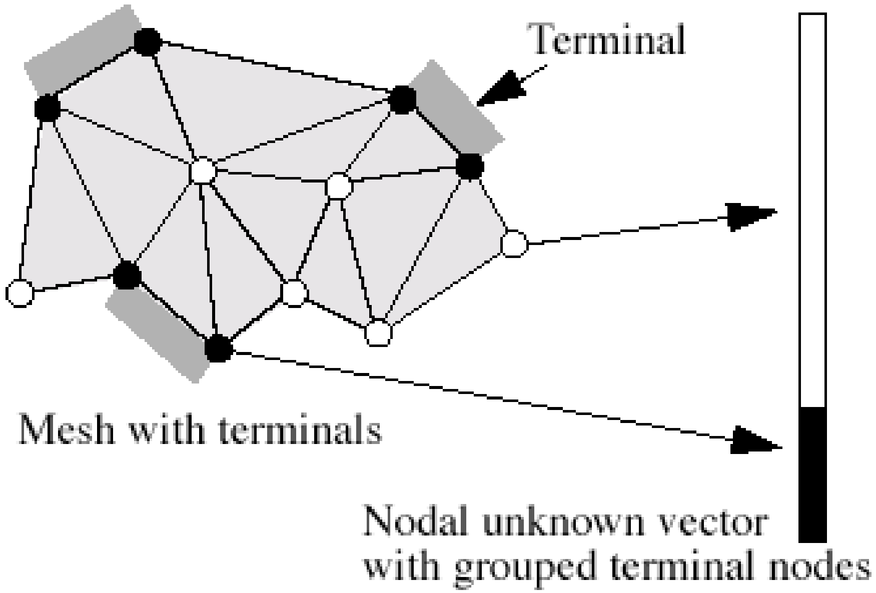
Homogeneous Heat Generation Rate



- Allows us to obtain after semidiscretization a system of ODE which can act as a black box.
- $$\begin{cases} \frac{d\boldsymbol{T}}{dt} = A \cdot \boldsymbol{T} + \boldsymbol{b}u \\ \boldsymbol{y} = C \cdot \boldsymbol{T} \end{cases}$$
- $u = I(t)^2 R(T)$
- Constant materials properties except resistivity bring forward a linear dynamic system (with small cheating).

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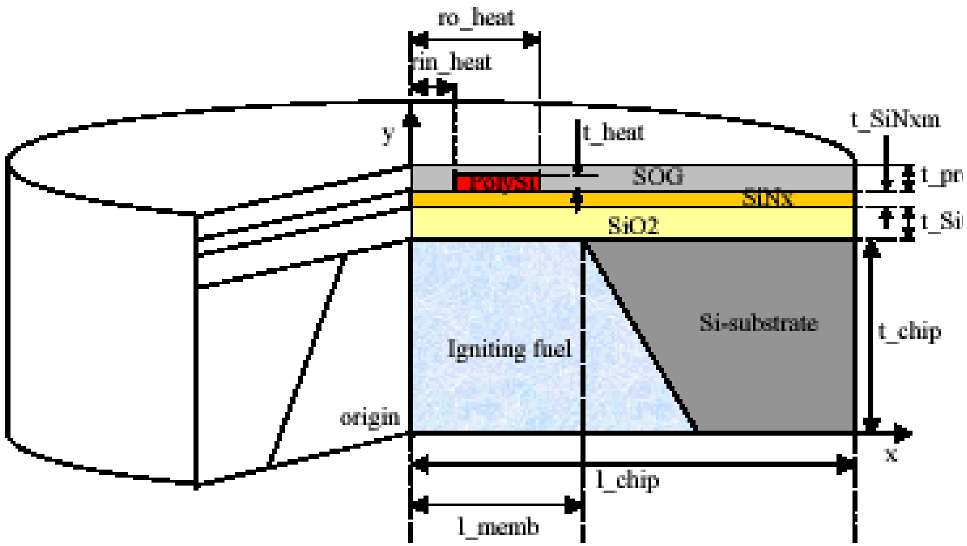
Conclusions

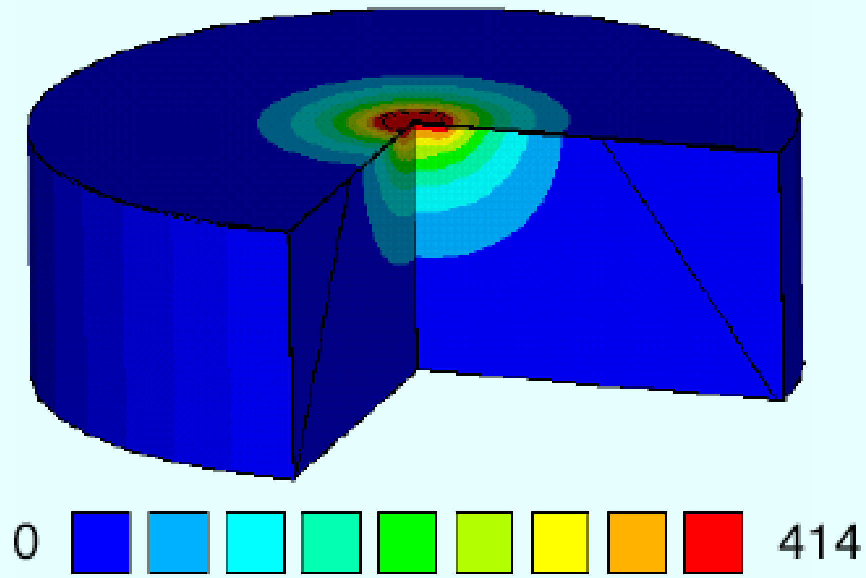
- Electro-thermal simulation can be can be easily linearized under homogeneous heat approximation.
- It perfectly fits the framework for model reduction.
- If the homogeneous heat generation hypothesis is not appropriate:
  - One can find a lumped representation of electric parts that contribute to the heat generation rate.
  - Assume that within each lumped element the heat generation rate is homogeneous.

2D Axisymmetrical Model

- Cross-section:



- Mesh with 1071 nodes (DoF).



Temperature distribution within the igniting wafer (2D axi-symmetric model) after 0.3s of heating with 80mW power; T_{ref} = 273K.

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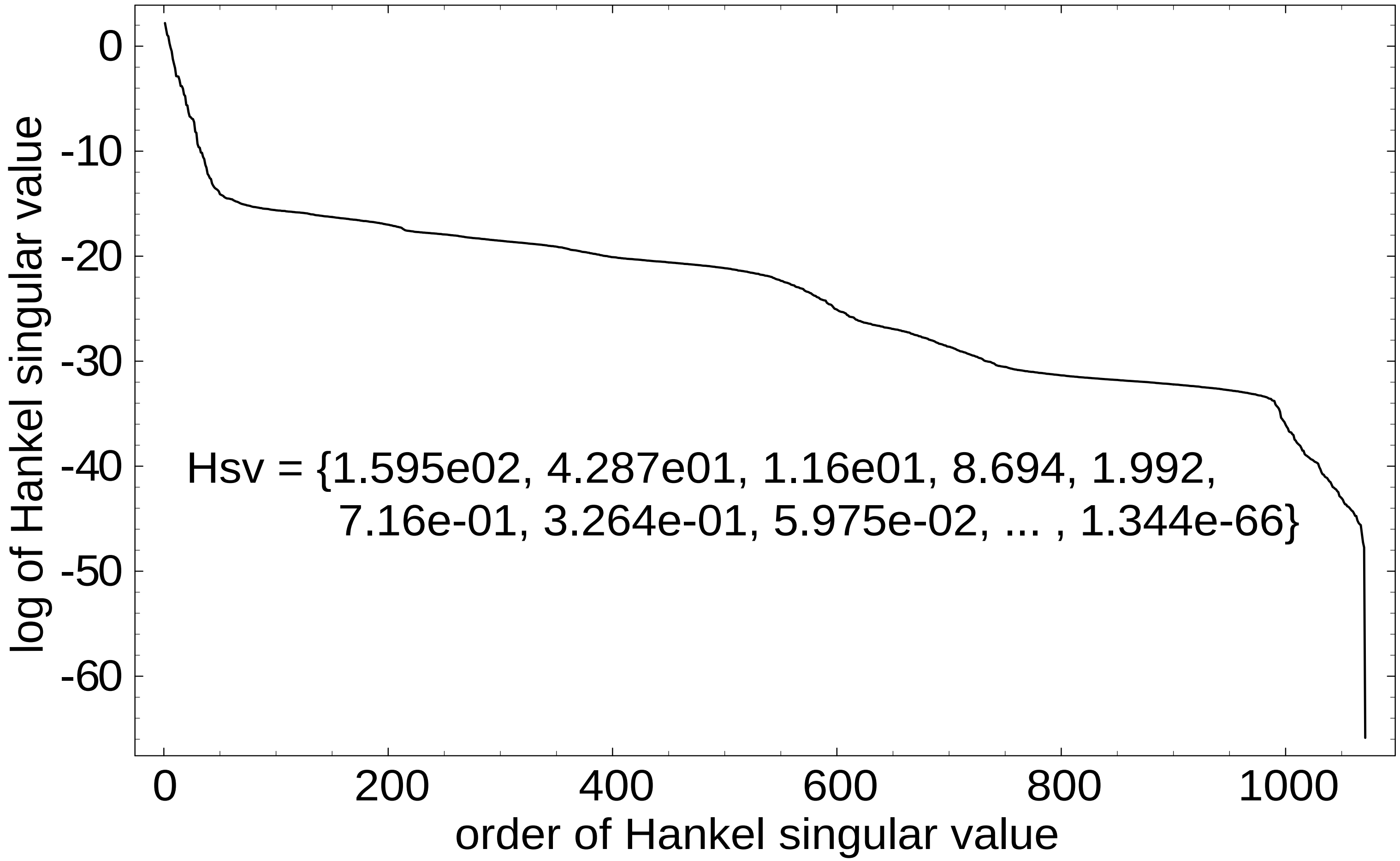
Decay of the Hankel singular values



Numerical Results

Comparison

- Control Theory:
  - Balanced Truncation Approximation,
- Moment Matching:
  - Arnoldi Algorithm,
- ANSYS dynamic condensation:
  - Guyan method.

Guyan method

- Based on Shur complement for stationary problem:

$$\begin{bmatrix} K_{ee} & K_{ei} \\ K_{ie} & K_{ii} \end{bmatrix} \begin{Bmatrix} T_e \\ T_i \end{Bmatrix} = \begin{Bmatrix} F_e \\ F_i \end{Bmatrix}$$

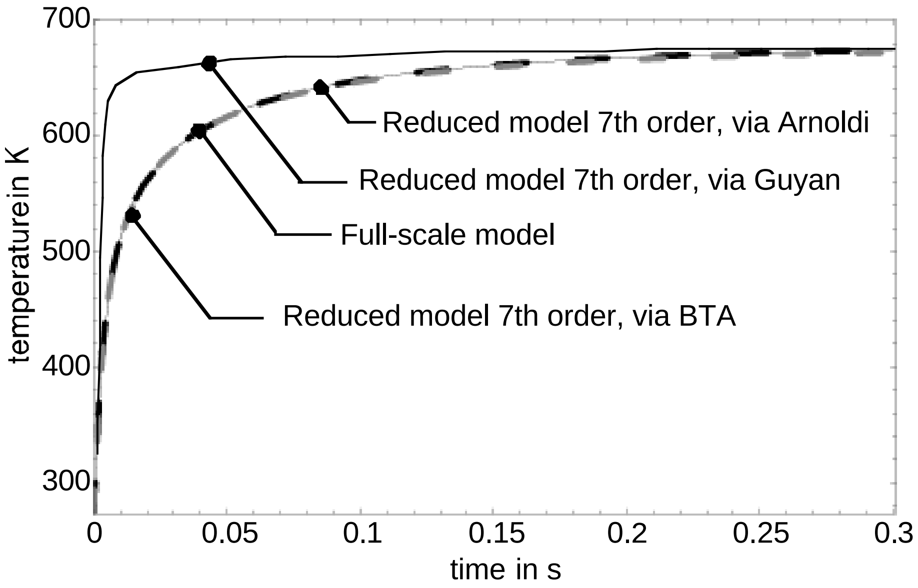
- Excluding internal nodes

$$[K_r] = [K_{ee}] - [K_{ei}][K_{ii}]^{-1}[K_{ie}]$$

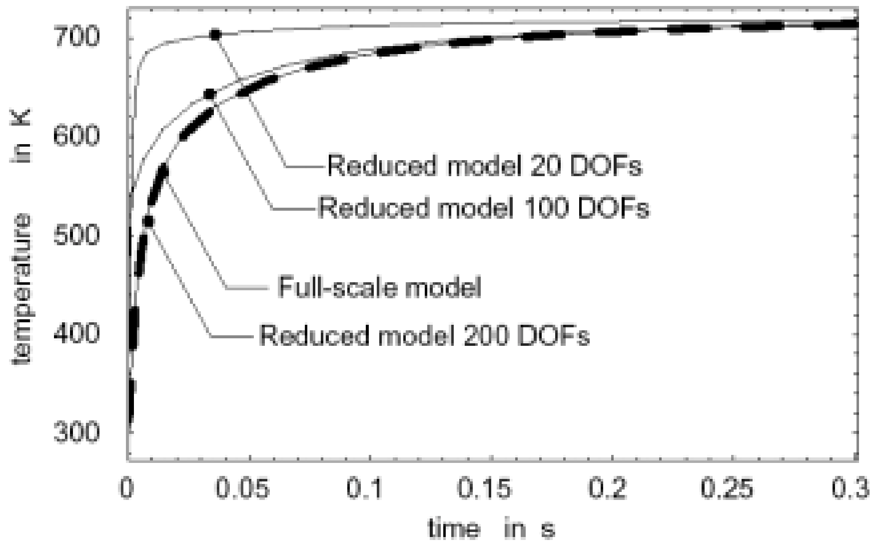
- Guyan suggested to use the same transformation for the mass matrix.
- ANSYS extended it for the damping matrix.
  - Automatic choice for the master DoF.

Comparison

- Solution of the full system (1071 order) and of the reduced systems for a single node.

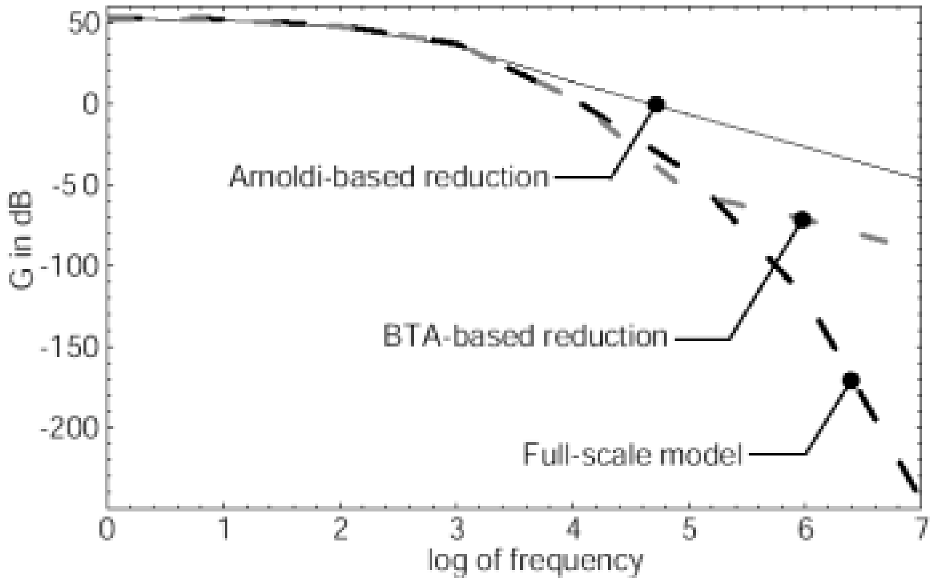


- Solution of the full system and of the reduced system for Guyan method.

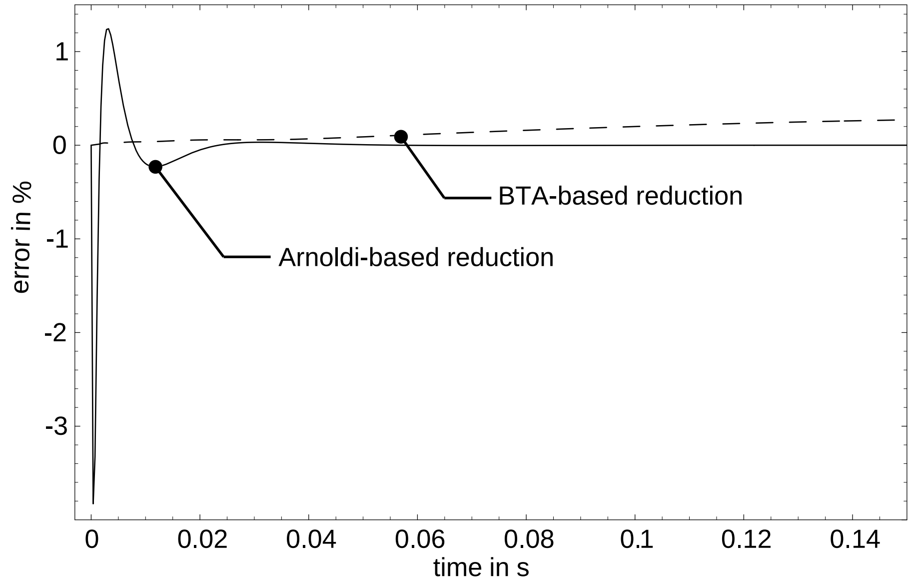


Comparison

- Transfer function for the full and reduced models for a single node.

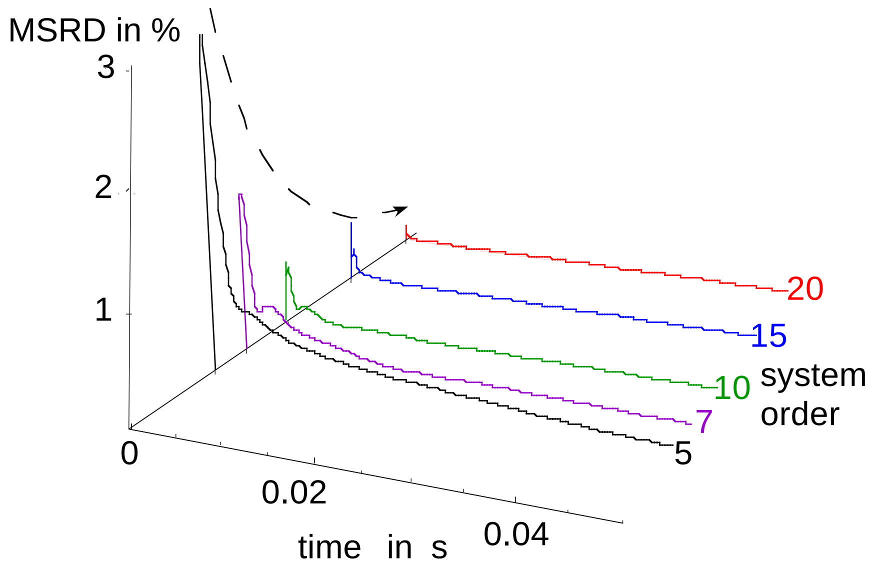


- Relative error corresponding to the plots during the initial 0.15 s.



Complete Output

- Arnoldi algorithm does not take into account outputs (matrix C) during model reduction.
- It allows us to find a low dimensional subspace for the whole state vector $\boldsymbol{x} = X \cdot \boldsymbol{z} + \boldsymbol{\varepsilon}$.
- This means that we can reproduce the complete output.
- The error of the reduced model for all the nodes in the case of the Arnoldi method.



Numerical Results

Conclusions

- Dimension of thermal part can be substantially reduced.
- Arnoldi algorithm is working quite well and can be recom-mended for the use in the case of electro-thermal simulations.
- Control theory methods are even better but they are not scaled to large systems.

Questions to research

- Stop criterion - how to choose the dimension of the reduced system.
- How to connect reduced thermal models to one another?
- Influence of nonlinear term in the input function.
- Nonhomogeneous heat generation.

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